



قسم هندسة الليزر والالكترونيات البصرية

Geometrical Optics

مرحلة ثانية

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Interference

Interference of light is the phenomena of multiple light waves interacting with one another under certain circumstances, One of the best examples of the interference of light is demonstrated by the light reflected from a film of oil floating on water. Another example is the soap bubble illustrated in Figure 1 that reflects a variety of beautiful colors when illuminated by natural or artificial light sources.

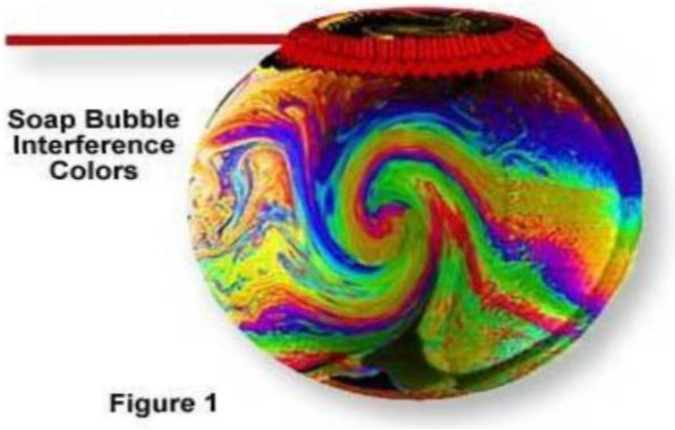


Figure 1

This dynamic interplay of colors derives from the simultaneous reflection of light from both the inside and outside surfaces of the bubble (Figure 1). The two surfaces are very close together (the bubble is only a few microns thick), and light reflected from the inner surface interferes both constructively and destructively with light reflected from the outer surface. This is because light reflected from the inner surface of the bubble must travel further than light reflected from the outer surface. When the light waves reflected from the inner and outer surface combine, they will interfere with each other, removing or reinforcing some parts of white light by destructive or constructive interference.If the extra distance traveled by the inner light waves is exactly the wavelength of the outer light waves, then they will recombine constructively, and bright colors of those wavelengths will be produced .

How Does the Interference of Light Occur?

Consider a pair of light waves from the same source that are traveling, for example, in direction D. If the vibrations are not in the same plane and are vibrating at 90 degrees to each other, then they cannot interfere with one another. This is the propagation direction (as illustrated in Figure 2), and if the vibrations (which are perpendicular to the propagation direction as represented by C in Figure 2) are parallel to each other and are also parallel with respect to the direction of vibration, then the light waves may interfere with each other. If the crests of one of the waves coincide with the crests of the other, the amplitudes are additive. If the amplitudes of both waves are equal, the resultant amplitude would be doubled. Bear in mind that light intensity varies directly as the square of the amplitude. Thus, if the amplitude is doubled, intensity is quadrupled. Such additive interference is called constructive interference (illustrated in Figure 2). If the crests of one wave coincide with the troughs of the other wave, the resulting amplitude is decreased or may even be completely canceled, as illustrated in Figure 3. This is called destructive interference. The result is a drop in intensity, or in the case of total cancellation, blackness.

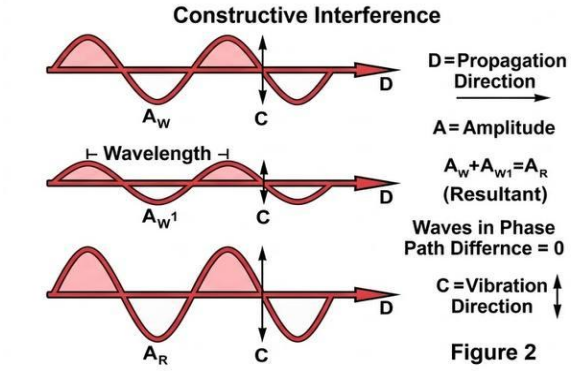


Figure 2

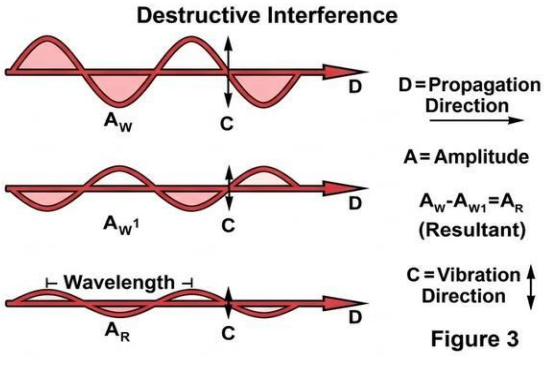


Figure 3

TWO-BEAM INTERFERENCE

We consider first the interference of two waves, represented by El and E2, where we take into account the vector property of the electric fields. In cases of interference, both waves typically originate from a single source and reunite after traveling along different paths. The direction of travel of the waves need not be the same when they come together, however, so that whereas they maintain the same frequency, they generally do not have the same propagation vector k.

$$\mathbf{E}_1 = \mathbf{E}_{01} \cos(\mathbf{k}_1 \cdot \mathbf{r} - \omega t + \epsilon_1)$$
$$\mathbf{E}_2 = \mathbf{E}_{02} \cos(\mathbf{k}_2 \cdot \mathbf{r} - \omega t + \epsilon_2)$$

At some general point P, defined by position vector **r**, the waves intersect to produce a disturbance whose electric field $\mathbf{E}_p$ is given by the principle of superposition,

$$\mathbf{E}_p = \mathbf{E}_1 + \mathbf{E}_2$$

-Measurement of the waves by their effect on the eye or some other light detector depends on the energy of the light beam.
-The radiant power density, or irradiance, E_e (W/m2), measures the time average of the square of the wave amplitude. Unfortunately, the standard symbol for irradiance

$$I = \epsilon_0 c \langle \mathbf{E}^2 \rangle$$
$$I = \epsilon_0 c \langle \mathbf{E}_1^2 + \mathbf{E}_2^2 + 2\mathbf{E}_1 \cdot \mathbf{E}_2 \rangle$$

the first two terms correspond to the irradiances of the individual waves, I_1 and I_2. The last term depends on an interaction of the waves and is called the interference term, I_{12}.

$$I = I_1 + I_2 + I_{12}$$

$$I_{12} = 2\epsilon_0 c \langle \mathbf{E}_1 \cdot \mathbf{E}_2 \rangle$$

$$\mathbf{E}_1 \cdot \mathbf{E}_2 = \mathbf{E}_{01} \cdot \mathbf{E}_{02} \cos(\mathbf{k}_1 \cdot \mathbf{r} - \omega t + \epsilon_1) \cos(\mathbf{k}_2 \cdot \mathbf{r} - \omega t + \epsilon_2)$$
$$\alpha \equiv \mathbf{k}_1 \cdot \mathbf{r} + \epsilon_1 \quad \text{and} \quad \beta \equiv \mathbf{k}_2 \cdot \mathbf{r} + \epsilon_2$$

$$\mathbf{E}_1 \cdot \mathbf{E}_2 = \mathbf{E}_{01} \cdot \mathbf{E}_{02} \cos(\alpha - \omega t) \cos(\beta - \omega t)$$

Expanding and multiplying the cosine factors, we arrive at

$$\langle \mathbf{E}_1 \cdot \mathbf{E}_2 \rangle = \mathbf{E}_{01} \cdot \mathbf{E}_{02} [\cos\alpha \cos\beta \langle \cos^2 \omega t \rangle + \sin\alpha \sin\beta \langle \sin^2 \omega t \rangle + (\cos\alpha \sin\beta + \sin\alpha \cos\beta) \langle \sin \omega t \cos \omega t \rangle]$$

$$\langle \cos^2 \omega t \rangle = \tfrac{1}{2}, \qquad \langle \sin^2 \omega t \rangle = \tfrac{1}{2}$$

$$\langle \sin \omega t \cos \omega t \rangle = 0$$

$$\langle \mathbf{E}_1 \cdot \mathbf{E}_2 \rangle = \tfrac{1}{2} \mathbf{E}_{01} \cdot \mathbf{E}_{02} \cos(\alpha - \beta)$$

$$\langle \mathbf{E}_1 \cdot \mathbf{E}_2 \rangle = \tfrac{1}{2} \mathbf{E}_{01} \cdot \mathbf{E}_{02} \cos[(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r} + (\epsilon_1 - \epsilon_2)]$$

where the expression in brackets is the phase difference between the two waves, E_1 and E_2

$$\delta = (\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r} + (\epsilon_1 - \epsilon_2)$$

Final Result: The time-averaged intensity I_{12} is derived and can be rewritten using the defined phase difference. $I_{12} = \epsilon_0 c \mathbf{E}_{01} \cdot \mathbf{E}_{02} \cos \delta$

$$I_1 = \epsilon_0 c \langle \mathbf{E}_1^2 \rangle = \tfrac{1}{2} \epsilon_0 c E_{01}^2 \qquad I_2 = \epsilon_0 c \langle \mathbf{E}_2^2 \rangle = \tfrac{1}{2} \epsilon_0 c E_{02}^2$$

In the case $E_{01} \parallel E_{02}$, their dot product in above equations is identical with the product of their magnitudes.

$$I_{12} = 2\sqrt{I_1 I_2} \cos \delta$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

Depending on whether $\cos \delta > 0$ or $\cos \delta < 0$ in above Eq. the interference term either augments or diminishes the sum of the individual irradiances I_1 and I_2, leading to constructive or destructive interference, respectively. On the other hand, if the initial phase difference (ϵ_1 - ϵ_2) in above Eq. varies randomly, the waves are said to be mutually incoherent, and $\cos \delta$ becomes a time-dependent factor whose average is zero.

To be more specific, when cos $\delta = +1$, constructive interference yields the maximum irradiance

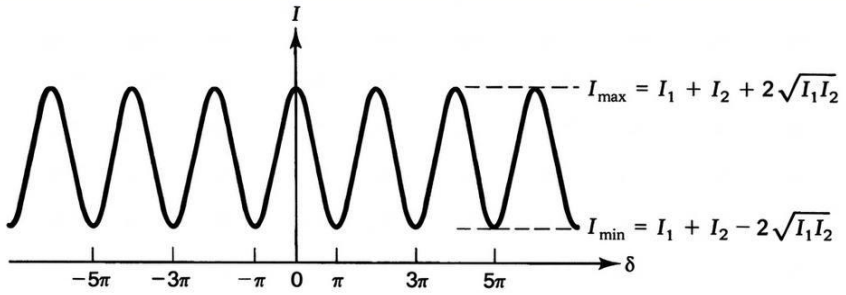
$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

This condition occurs whenever the phase difference $\delta = 2m\pi$, where m is any integer or zero. On the other hand, when cos $\delta = -1$, destructive interference yields the minimum, or background, irradiance

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

a condition that occurs whenever $\delta = (2m+1)\pi$. A plot of irradiance I versus phase δ in Figure below, exhibits periodic fringes. Destructive interference is complete, that is, cancelation is complete, when $I_1 = I_2 = I_0$.

$$I_{max} = 4I_0 \quad \text{and} \quad I_{min} = 0$$



A measure of fringe contrast, also called visibility, with values between 0 and 1, is given by the

$$\text{fringe contrast} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Example

Two interfering beams with parallel electric fields are given by

$$E_1 = 2\cos\left(\mathbf{k}_1 \cdot \mathbf{r} - \omega t + \frac{\pi}{3}\right) \quad \text{(kV/m)}$$

$$E_2 = 5\cos\left(\mathbf{k}_2 \cdot \mathbf{r} - \omega t + \frac{\pi}{4}\right) \quad \text{(kV/m)}$$

Let us determine the irradiance contributed by each beam acting alone and that due to their mutual interference at a point where their path difference is zero. We have

$$I_1 = \tfrac{1}{2}\epsilon_0 c E_{01}^2 = \tfrac{1}{2}\epsilon_0 c(2000)^2 = 5309 \text{ W/m}^2$$

$$I_2 = \tfrac{1}{2}\epsilon_0 c E_{02}^2 = \tfrac{1}{2}\epsilon_0 c(5000)^2 = 33{,}180 \text{ W/m}^2$$

$$I_{12} = 2\sqrt{I_1 I_2}\cos\delta = 2\sqrt{(5309 \times 33180)}\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = 25{,}640 \text{ W/m}^2$$

To find the fringe contrast near the region of superposition we must calculate

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = 5309 + 33180 + 2\sqrt{5309 \times 33180}$$
$$= 65{,}034 \text{ W/m}^2$$

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = 5309 + 33180 - 2\sqrt{5309 \times 33180}$$
$$= 11{,}945 \text{ W/m}^2$$

The contrast is then given by $(I_{\max} - I_{\min})/(I_{\max} + I_{\min})$, or

$$\text{fringe contrast} = \frac{65{,}034 - 11{,}945}{65{,}034 + 11{,}945} = 0.690$$