قسم هندسة الليزر والالكترونيات البصرية

Geometrical Optics

مرحلة ثانية

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spherical thick lens

It is, a lens whose thickness along its optical axis cannot be ignored without leading to serious errors in analysis. when a lens moves from the category of thin to thick clearly depends on the accuracy required.

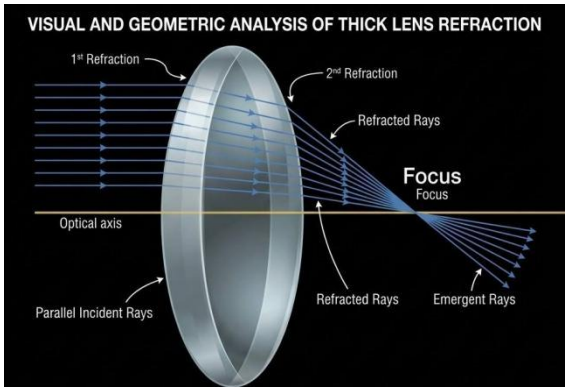
The thick lens can be treated as glass medium bounded by two spherical refracting surfaces. The image of a given object, formed by refraction at the first surface, becomes the object for refraction at the second surface.

the object distance for the second surface takes into account the thickness of the lens. The image formed by the second surface is then the final image due to the action of the composite thick lens.

There are six cardinal points on the axis of a thick lens:

from this point the imaging system properties can be deduced. Planes' normal to the axis at these

points are called the cardinal planes.



1. the first and second system focal points (F1and F2).
2. the first and second principal points (H1 and H2).
3. the first and second nodal points (N1 and N2).

A ray from the first focal point F1 is rendered parallel to the axis and a ray parallel to the axis is refracted by the lens through the second focal point F2

the focal length f_1 is given by $$\frac{1}{f_1} = \frac{n_L - n'}{nR_2} - \frac{n_L - n}{nR_1} - \frac{(n_L - n)(n_L - n')}{nn_L}\frac{t}{R_1 R_2}$$

The focal length f_2 is given by $$f_2 = -\frac{n'}{n} f_1$$

The principal planes can be located next using

$$r = \frac{n_L - n'}{n_L R_2} f_1 t \quad \text{and} \quad s = -\frac{n_L - n}{n_L R_1} f_2 t$$

The positions of the nodal points are given by

$$v = \left(1 - \frac{n'}{n} + \frac{n_L - n'}{n_L R_2} t\right) f_1 \quad \text{and} \quad w = \left(1 - \frac{n}{n'} - \frac{n_L - n}{n_L R_1} t\right) f_2$$

Image and object distances and lateral magnification are related by

$$-\frac{f_1}{s_o} + \frac{f_2}{s_i} = 1 \quad \text{and} \quad m = -\frac{n s_i}{n' s_o}$$

as long as the distances s_o and s_i, as well as focal lengths, are measured relative to corresponding principal planes. In the ordinary case of a lens in air, with n = n' = 1, notice that r = v and s = w.

First and second principal points are superimposed over corresponding nodal points. Also, first and second focal lengths are equal in magnitude, and the equations.

$$-\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \quad \text{and} \quad m = -\frac{s_i}{s_o}$$

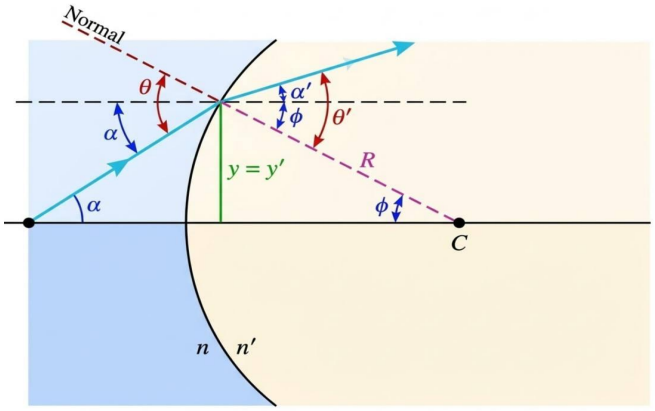
Example

Determine the focal lengths and the principal points for a 4-cm thick, biconvex lens with refractive index of 1.52 and radii of curvature of 25 cm, when the lens caps the end of a long cylinder filled with water (n = 1.33).

The refraction matrix

Consider next the refraction of a ray at a spherical surface separating media of refractive indices n and n', as shown in Figure. We need to relate the ray coordinates (y ', α ') after refraction to those before refraction, (y, α). Since refraction occurs at a point, there is no change in elevation, and y = y'.

The angle α', on the other hand, is by inspection of Figure



$$y' = (1)y + (0)\alpha$$

$$\alpha' = \left[\left(\frac{1}{R}\right)\left(\frac{n}{n'} - 1\right)\right]y + \left(\frac{n}{n'}\right)\alpha$$

In matrix form

$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n}{n'} - 1\right) & \frac{n}{n'} \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

The reflection matrix

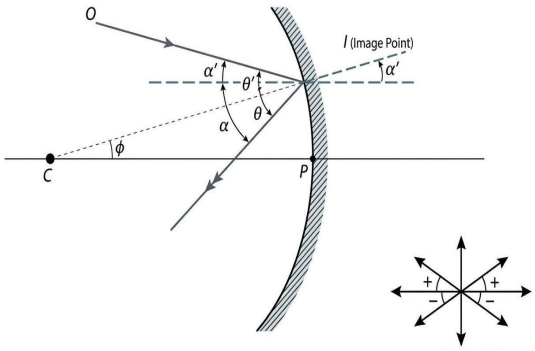
consider reflection at a spherical surface, illustrated in Figure below. In the case considered, a concave mirror, R is negative. We need to add a sign convention for the angles that describe the ray directions. From the geometry of Figure , with both α and α' positive,

$$\alpha = \theta + \phi = \theta + \frac{y}{-R}$$

$$\alpha' = \theta' - \phi = \theta' - \frac{y}{-R}$$

Using these relations together with the law of reflection, $\theta = \theta'$,

$$\alpha' = \theta' + \frac{y}{R} = \theta + \frac{y}{R} = \alpha + \frac{2y}{R}$$



the two desired linear equations of the spherical reflection surface are

$$y' = (1)y + (0)\alpha$$

$$\alpha' = \left(\frac{2}{R}\right)y + (1)\alpha$$

In matrix form,

$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

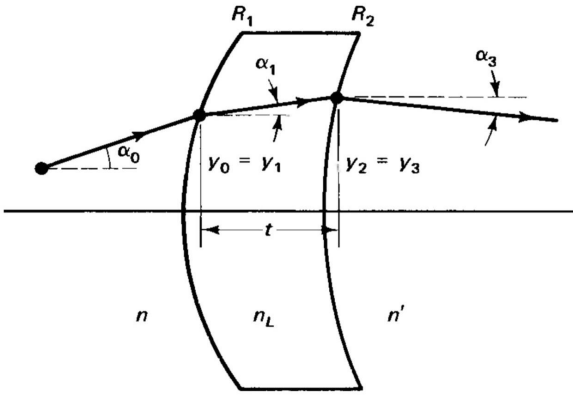
Thick-lens and thin-lens matrices

a matrix that represents the action of a thick lens on a ray of light was constructed. For generality, we assume different media on opposite sides of the lens, having refractive indices n and n', as shown in Figure below. In traversing the lens, the ray undergoes two refractions and one translation, steps for which we have already derived matrices.

where we have chosen for simplicity a lens with positive radii of curvature, we may write, symbolically

$$\begin{bmatrix} y_1 \\ \alpha_1 \end{bmatrix} = M_1 \begin{bmatrix} y_0 \\ \alpha_0 \end{bmatrix} \quad \text{for the first refraction}$$

$$\begin{bmatrix} y_2 \\ \alpha_2 \end{bmatrix} = M_2 \begin{bmatrix} y_1 \\ \alpha_1 \end{bmatrix} \quad \text{for the translation}$$



$$\begin{bmatrix} y_3 \\ \alpha_3 \end{bmatrix} = M_3 \begin{bmatrix} y_2 \\ \alpha_2 \end{bmatrix} \quad \text{for the second refraction}$$

Telescoping these matrix equations results in

$$\begin{bmatrix} y_3 \\ \alpha_3 \end{bmatrix} = M_3 M_2 M_1 \begin{bmatrix} y_0 \\ \alpha_0 \end{bmatrix}$$

Generalizing, the matrix equation representing any number N of translations, reflections, and refractions is given by

$$\begin{bmatrix} y_f \\ \alpha_f \end{bmatrix} = M_N M_{N-1} \cdots M_2 M_1 \begin{bmatrix} y_0 \\ \alpha_0 \end{bmatrix}$$

the ray-transfer matrix representing the entire optical system is

$$M = M_N M_{N-1} \cdots M_2 M_1$$

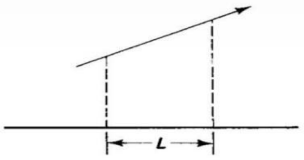
When multiplied together,

$$M = \begin{bmatrix} 1 & 0 \\ \frac{n_L - n}{n}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) & 1 \end{bmatrix} \quad \Rightarrow \quad M = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

$$\frac{1}{f} = \frac{n_L - n}{n}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

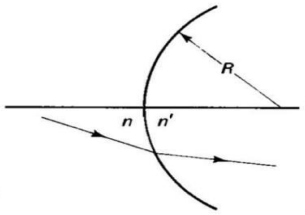
Translation matrix:

$$M = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$



Refraction matrix, spherical interface:

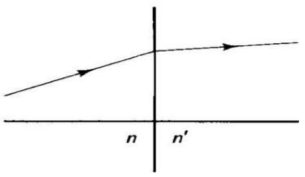
$$M = \begin{bmatrix} 1 & 0 \\ \frac{n - n'}{Rn'} & \frac{n}{n'} \end{bmatrix}$$



(+R) : convex
(−R) : concave

Refraction matrix, plane interface:

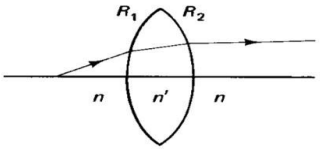
$$M = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n}{n'} \end{bmatrix}$$



Thin-lens matrix:

$$M = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

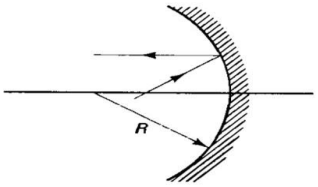
$$\frac{1}{f} = \frac{n' - n}{n}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



(+f) : convex
(−f) : concave

Spherical mirror matrix:

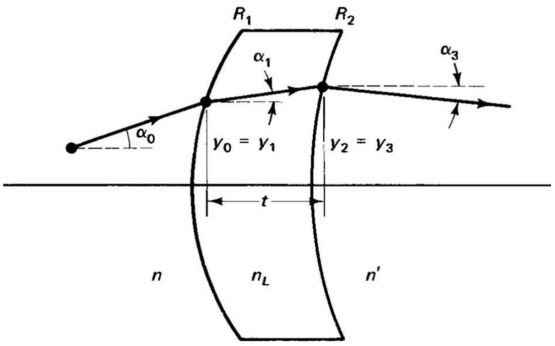
$$M = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$



(+R) : convex
(−R) : concave

Example

Find the system matrix for the thick lens specify by choosing R1 = 45 cm, R2 = 30 cm, t = 5 cm, nL = 1.60, and n = n' = 1.



(H.M)

A biconvex lens of 5 cm thickness and index 1.60 has surfaces of radius 40 cm. If this lens is used for objects in water, with air on its opposite side, determine its effective focal length and sketch its focal and principal points.