جامعة اشور
كلية الهندسة
قسم الهندسة المدنية

(ENGINEERING STATISTICS)

الاحصاء الهندسي

المرحلة الأولى

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(The Quantiles)

The quartiles, deciles and percentiles for grouped data can be found by using the following expressions:

$$Q = a + \left[\frac{q - n_1}{f_q}\right].C$$

$$D = a + \left[\frac{d - n_1}{f_d}\right].C$$

$$P = a + \left[\frac{p - n_1}{f_p}\right].C$$

Where:

a = the real lower limit of quartiles, deciles or percentiles interval.

n_1 = the cumulative frequency of the previous interval of the quartiles deciles or percentiles interval.

f_q, f_d, and f_p = the frequency of quartiles, deciles and percentiles interval respectively.

C = the length of the interval.

Example 34: Find the quartiles Q1, Q2, and Q3 of the data shown in example 33.

Solution:

1- find the cumulative frequency and the summation of frequencies and real interval limit.

Class interval	Frequency (f_i)	Cumulative Frequency	Real Interval Limit
50-69	3	3	49.5-69.5
70-89	7	10	69.5-89.5
90-109	4	14	89.5-109.5
110-129	4	18	109.5-129.5
130-149	9	27	129.5-149.5
	$\Sigma f_i=27$		

2- Find the arrangement number of quartiles to find quartile interval 1.

q1 = (1/4) x n = (1/4) x 27 = 6.75

The interval of quartile number 1 has the cumulative frequency = 10

$$Q1 = a + \left[\frac{q1 - n_1}{f_q}\right].C$$

$a = 69.5$, $q_1 = 6.75$, $n_1 = 3$, $f_q = 7$, $C = 20$

$$Q1 = 69.5 + \left[\frac{6.75 - 3}{7}\right].20 = 80.2$$

3- find the arrangement number of quartiles to find quartile interval 2.

q2 = (2/4) x n = (2/4) x 27 = 13.5

The interval of quartile number 2 has the cumulative frequency = 14

$$Q_2 = 89.5 + \left[\frac{13.5 - 10}{4}\right].20 = 107$$

4- find the arrangement number of quartiles to find quartile interval 3.

q3 = (3/4) x n = (3/4) x 27 = 20.25

The interval of quartile number 3 has the cumulative frequency = 27

$$Q_3 = 129.5 + \left[\frac{20.25 - 18}{9}\right].20 = 134.5$$

Example 35: find the deciles D1, D5, and D9 of the data shown in example 34.

1- find the cumulative frequency and the summation of frequencies and real interval limit.

Class interval	Frequency (f_i)	Cumulative Frequency	Real Interval Limit
50-69	3	3	49.5-69.5
70-89	7	10	69.5-89.5
90-109	4	14	89.5-109.5
110-129	4	18	109.5-129.5
130-149	9	27	129.5-149.5
	$\Sigma f_i=27$		

2) find the arrangement number of deciles to find deciles interval D1.

d1 = (1/10) x n = (1/10) x 27 = 2.7

The interval of deciles number 1 has the cumulative frequency = 3

$$D1 = 49.5 + \left[\frac{2.7-0}{3}\right].20 = 67.5$$

3) find the arrangement number of deciles to find deciles interval D5.

d5 = (5/10) x n = (5/10) x 27 = 13.5

The interval of deciles number 5 has the cumulative frequency = 14

$$D5 = 89.5 + \left[\frac{13.5-10}{4}\right].20 = 107$$

4) find the arrangement number of deciles to find deciles interval D9.

d9 = (9/10) x n = (9/10) x 27 = 24.3

The interval of deciles number 9 has the cumulative frequency = 27

$$D9 = 129.5 + \left[\frac{24.3-18}{9}\right].20 = 143.5$$

Example 36: find the percentiles P8, P50, and P85 of the data shown in example 34.

1- find the cumulative frequency and the summation of frequencies and real interval limit.

Class interval	Frequency (f_i)	Cumulative Frequency	Real Interval Limit
50-69	3	3	49.5-69.5
70-89	7	10	69.5-89.5
90-109	4	14	89.5-109.5
110-129	4	18	109.5-129.5
130-149	9	27	129.5-149.5
	$\Sigma f_i=27$		

2) find the arrangement number of percentiles to find percentiles interval P8.

p8 = (8/100) x n = (8/100) x 27 = 2.16

The interval of percentiles number 8 has the cumulative frequency = 3

$$P8 = 49.5 + \left[\frac{2.16-0}{3}\right].20 = 63.9$$

3) find the arrangement number of percentiles to find percentiles interval P50.

p50 = (50/100) x n = (50/100) x 27 = 13.5

The interval of percentiles number P50 has the cumulative frequency = 14

$$P50 = 89.5 + \left[\frac{13.5 - 10}{4}\right].20 = 107$$

4) find the arrangement number of percentiles to find percentiles interval P85.

p85 = (85/100) x n = (85/100) x 27 = 22.95

The interval of percentiles number P85 has the cumulative frequency = 27

$$P85 = 129.5 + \left[\frac{22.95 - 18}{9}\right].20 = 140.5$$

Example 37: Find Q2, D6, P65 for the data of example 31

q2 = (2/4) x n = (2/4) x 100 = 50

The interval of quantiles number Q2 has the cumulative frequency = 65

$$Q2 = 65.5 + \frac{50-23}{42} * 3 = 67.42$$

d6= (6/10) x n = (6/10) x 100 = 60

The interval of deciles number D6 has the cumulative frequency = 65

$$D6 = 65.5 + \frac{60-23}{42} * 3 = 68.14$$

P65= (65/100) x n = (65/100) x 100 = 65

The interval of percentiles number P65 has the cumulative frequency = 65

$$P65 = 65.5 + \frac{65-23}{42} * 3 = 68.5$$

Classes	fi	Midpoint xi	Cumulative frequency
60-62	5	61	5
63-65	18	64	23
66-68	42	67	65
69-71	27	70	92
72-74	8	73	100
	$\sum = 100$		

(Calculating Measures of Variation with Frequency Data)

-The Mean Deviation

If the data are given in a frequency distribution form the mean deviation becomes:

$$M.D = \frac{\sum_{i=1}^{n} |x_i - \bar{x}| * f_i}{\sum_{i=1}^{n} f_i}$$

Where: xi: the midpoints of classes; fi: the frequency for each class; n: the number of classes.

Example 38: Find the mean deviation for the data shown in example 31:

Solution:

1- find the midpoint xi

2- find the mean $\bar{X} = \frac{\sum_{i=1}^{k} xi\ fi}{\sum_{i=1}^{k} fi} = 6745/100 = 67.45$

3- find the mean deviation

$$M.D = \frac{\sum_{i=1}^{n} |x_i - \bar{x}| * f_i}{\sum_{i=1}^{n} f_i}$$

M.D = 226.5/100 = 2.265

Class	Frequency f^i	Midpoint xi	fi* xi	$\|xi - \bar{x}\|$	$\|xi - \bar{x}\|$fi
60 – 62	5	61	305	6.45	32.25
63 – 65	18	64	1152	3.45	62.1
66 – 68	42	67	2814	0.45	18.9
69 – 71	27	70	1890	2.55	68.85
72 – 74	8	73	584	5.55	44.4
	$\sum fi = 100$		6745		226.5

(The Variance, the Standard Deviation and coefficient of variation for classified data)

$$S^2 = \frac{\sum_{i=1}^{k} (xi - \bar{x})^2 fi}{\sum_{i=1}^{k} fi},$$

$$S = \sqrt{S^2} = \sqrt{\frac{\sum_{i=1}^{k} (xi - \bar{x})^2 fi}{\sum_{i=1}^{k} fi}}$$

$$C.V = \frac{S}{\bar{X}}.100\%$$

Where:
xi: the midpoints of classes;
fi: the frequency for each class;
k: the number of classes.

Example 39: Use the data shown in example 38 to find the variance, standard deviation and coefficient of variation:

Solution:

From example 38, the mean is 67.45

Class	Frequency f_i	xi	$(xi - \bar{x})^2 fi$
60 – 62	5	61	208.01
63 – 65	18	64	214.25
66 – 68	42	67	8.51
69 – 71	27	70	175.57
72 – 74	8	73	246.42
	$\sum fi = 100$		852.76

$$S^2 = \frac{\sum_{i=1}^{k} (xi - \bar{x})^2 fi}{\sum_{i=1}^{k} fi} = \frac{852.76}{100} = 8.53 \qquad S = \sqrt{S^2} = \sqrt{8.53} = 2.92$$

$$C.V = (2.92 / 67.45) * 100 = 4.33$$

Homework No. 4

1- Suppose that the following data represent the results of compressive strength for cubes after 28 days of curing. Construct a grouped frequency distribution for the data and find:
(a) Relative Frequency, (b) percentage frequency, (c) cumulative frequency, (d) construct the Histogram to represent the data
49, 57, 38, 73, 81, 74, 59, 76, 65, 69, 54, 56, 69, 68, 78, 85, 49, 69, 61, 48, 81, 68, 37, 43
78, 82, 43, 64, 67, 52, 56, 81, 77, 79, 85, 40, 85, 59, 80, 60.

2- For the data shown below, construct a frequency distribution and find:
(a) the arithmetic mean (b) the geometric mean, (c) the harmonic mean, (d) the median (e) the mode (f) constructs the frequency polygon to represent the data
12, 17, 12, 14, 16, 18, 16, 18, 16, 17, 15, 16, 12, 15, 16, 12, 14, 15, 12, 15, 13, 16, 18, 16.

3- For the data shown below, construct a frequency distribution and find:
(a) the mean deviation, (b) the standard deviation, (c) Q2, D7, P50, P77, (d) construct the cumulative frequency graph to represent the data.
164, 225, 225, 140, 188, 210, 238, 146, 201, 544, 550, 188, 415, 261, 164, 478, 684, 330, 307, 435, 857, 183, 381, 275, 578, 450, 385, 297, 390, 515.