جامعة اشور
كلية الهندسة
قسم الهندسة المدنية

(ENGINEERING STATISTICS)

الاحصاء الهندسي

المرحلة الأولى

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المحاضرة السابعة

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(Calculating Measures of Location with Frequency Data)

- The Arithmetic Mean

It is denoted by ($\bar{x}$) and calculated by:

Where:

$$\bar{X}=\frac{\sum_{i=1}^{n} X_i f_i}{\sum_{i=1}^{n} f_i}$$

xi: the midpoint of each class; fi: the frequency of each class; n: number of classes.

Example 29: At the site, the length of several piles is as follows. determine the arithmetic mean.

Pile Length (x_i)	3	7	8	12	19	21	25	31	38	45	52	60	68	74	
Frequency (f_i)	2	3	1	5	4	2	1	4	6	1	5	6	3	4	$\Sigma f_i = 47$

Solution:

The arithmetic mean is calculated from the following expression:

$$\bar{X}=\frac{\sum_{i=1}^{n} X_i f_i}{\sum_{i=1}^{n} f_i}$$

Xi * fi	6	21	8	60	76	42	25	124	228	45	260	360	204	296	$\Sigma X_i * f_i = 1755$

$$\bar{X} = \frac{1755}{47} = 37.34$$

Example 30: Calculate the arithmetic mean of the vehicle speeds (km/h) from the table below:

Class	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80	81 – 90	91 – 100
Frequency (f_i)	1	2	5	15	25	20	12

Solution:

Class	Frequency (fi)	Centre of class or midpoints (xi)	xi * fi
31 – 40	1	35.5	35.5
41 – 50	2	45.5	91
51 – 60	5	55.5	277.5
61 – 70	15	65.5	982.5
71 – 80	25	75.5	1887.5
81 – 90	20	85.5	1710
91 – 100	12	95.5	1146
	$\sum fi = 80$		$\sum xifi = 6130$

$$\bar{X} = \frac{\sum_{i=1}^{k} xi\ fi}{\sum_{i=1}^{k} fi} = \frac{6130}{80} = 76.63\ km/h$$

- ***The Geometric Mean***

$$\bar{X}_g = (x_1{}^{f1} * x_2{}^{f2} * x_3{}^{f3} \dots\dots\dots * x_k{}^{fk})^{1/n}$$

or

$$\log \bar{X}_g = \frac{1}{n} \sum_{i=1}^{k} fi \ \log xi$$

Where: $\bar{X}_g$ is the geometric mean; $x_1, x_2 \cdots x_k$ are centers of classes; fi is the frequency; k is the number of classes, n is the number of readings.

Example 31: Find the geometric mean for the following table.

Class	Frequency (f_i)
60 – 62	5
63 – 65	18
66 – 68	42
69 – 71	27
72 – 74	8

Solution:

Class	Frequency (fi)	a- Midpoints (xi)
60 – 62	5	61
63 – 65	18	64
66 – 68	42	67
69 – 71	27	70
72 – 74	8	73
	$\sum fi = 100$	

$$\bar{X}_g = (x_1^{f1} * x_2^{f2} * x_3^{f3} \dots \dots \dots * x_k^{fk})^{1/n}$$

Or $$log\,\bar{X}_g = \frac{1}{n}\sum_{i=1}^{k} fi\ log\ xi$$

$$\bar{X}_g = (61^5 * 64^{18} * 67^{42} * 70^{27} * 73^8)^{1/100} = 67.38$$

Midpoint (x_i)	Frequency (f_i)	Log xi	Log Xi * fi
61	5	1.785	8.925
64	18	1.806	32.508
67	42	1.826	76.692
70	27	1.845	49.815
73	8	1.863	14.904
	Σ fi = 100		Σ fi *Log xi = 182.844

$$log\bar{X}_g = \frac{1}{100} * 182.844 = 1.82844,$$

$$\bar{X}_g = 10^{1.828} = 67.38$$

- ***The Harmonic Mean***

$$\bar{X}_h = \frac{\sum_{i=1}^{k} fi}{\sum_{i=1}^{k} fi/xi}$$

Where:

$\bar{x}_h$ is the harmonic mean; xi, midpoints of classes; fi, the frequency; k, number of classes.

Example 32: Find the harmonic mean for the data showed in example 31:

Midpoint (x_i)	Frequency (f_i)	f_i/x_i
61	5	0.082
64	18	0.281
67	42	0.627
70	27	0.386
73	8	0.109
	Σ fi = 100	Σ f_i/x_i= 1.485

$$\bar{X}_h = \frac{\sum_{i=1}^{k} fi}{\sum_{i=1}^{k} fi/xi} = 100/1.485 = 67.34$$

- ***The Median***

The median interval is the first interval that have a cumulative frequency equal or higher than the half of total frequency.

$$M = a + \frac{n/2 - n_1}{fm} * \Delta$$

Where:

M, the median; a, the lower-class boundary of the median class; n, number of items in the data (i.e., total frequency); n_1, sum of frequencies of all classes lower than the median class; f_m, frequency of the median class; Δ, class interval.

Example 33: Find the median for the data shown in Table.

Class interval	Frequency (f_i)
50-69	3
70-89	7
90-109	4
110-129	4
130-149	9

Solution:

Find the cumulative frequency and the summation of frequencies:

Class interval	Frequency (f_i)	Cumulative Frequency
50-69	3	3
70-89	7	10
90-109	4	14
110-129	4	18
130-149	9	27
	Σf_i=27	

n = 27

n/2 = 13.5

The median interval becomes: 90 - 109. Because its cumulative frequency is (14), and this is the first greater or equal frequency of (n/2)=13.5.

$$M = a + \frac{n/2 - n_1}{fm} * \Delta$$

a= 89.5, n=27, n_1=10, f_m=4, Δ = 20

$$M = 89.5 + \frac{\frac{27}{2} - 10}{4} * 20 = 107$$