Digital Systems

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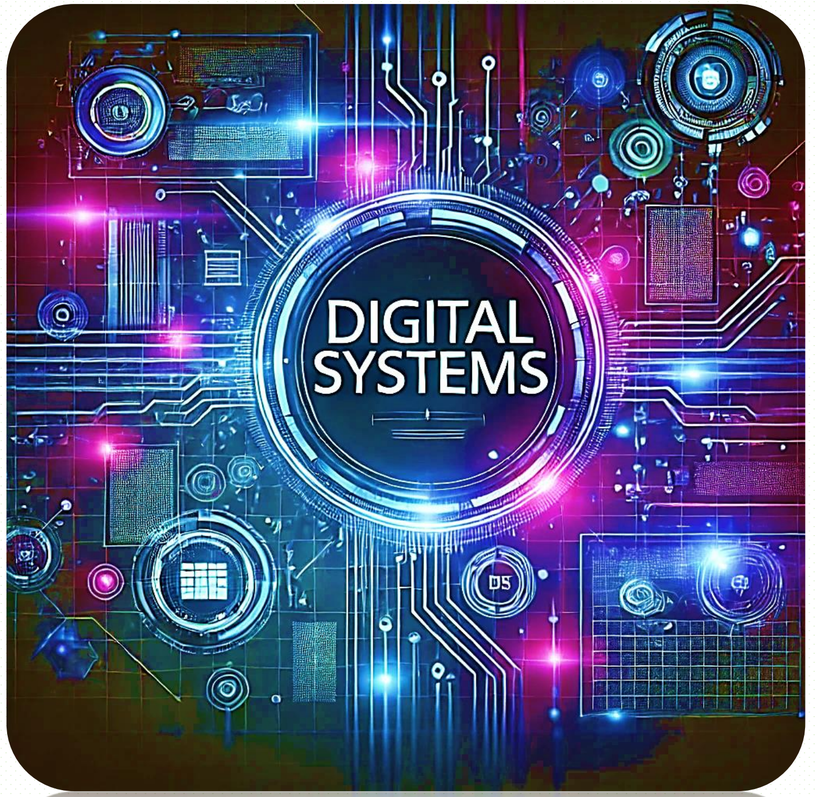


Computer Techniques Engineering Department

(K-Map)

Simplify the Boolean Function using Karnaugh Map

- The second method that used to simplify the Boolean function is the Karnaugh map. K-map basically deals with the technique of inserting the values of the output variable in cells within a rectangle or square grid according to a definite pattern. The number of cells in the K-map is determined by the number of input variables and is mathematically expressed as two raised to the power of the number of input variables, i.e., 2^n, where the number of input variables is n.
- Thus, to simplify a logical expression with **two inputs**, we require a K-map with (2^2 = 4) cells. A **four-input** logical expression would lead to a (2^4 = 16) celled-K-map, and so on.

(K-Map)

- Karnaugh map (K-map)
  - A visualization tool for logic simplification. It is a flattened version of Boolean cube that allows you to solve even difficult and complex Boolean expression into the minimized form. A convenient tool for you to know for up to 6 input variable expressions (e.g. X = f(A, B, C, D, E, F))
- Don’t cares
  - Sometimes the logic output doesn’t matter. When we don’t care if the output is 0 or 1, rather than assigning random outputs, it is best to denote it as “Don’t care.” If you learn how to use the “don’t care’s”, you will be able to build even more efficient circuits than without them.

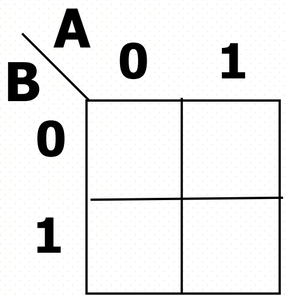
Advantages of K-Maps

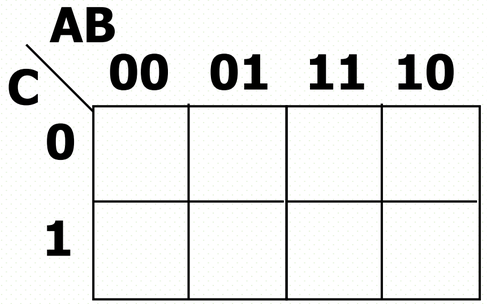
- 1- The K-map simplification technique is simpler and less error-prone compared to the method of solving the logical expressions using Boolean laws.
- 2- It prevents the need to remember each and every Boolean algebraic theorem.
- 3- It involves fewer steps than the algebraic minimization technique to arrive at a simplified expression.
- 4- K-map simplification technique always results in minimum expression if carried out properly.

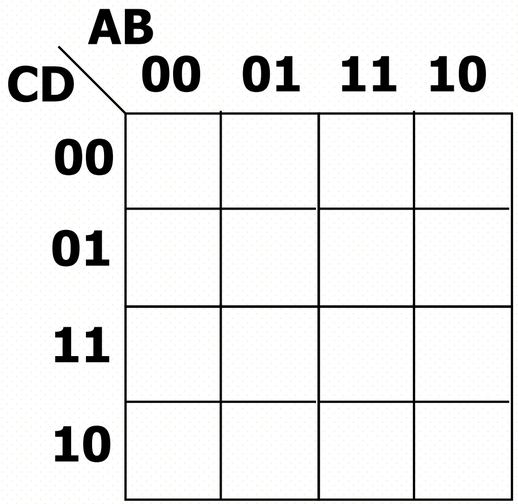
K-mapping & Minimization Steps

- Step 1: generate K-map based on the number of input variables n
  Step 2: group all adjacent 1s without including any 0s. All groups must be rectangular and contain a “power-of-2” number of 1s 1, 2, 4, 8, 16, 32, …
  Step 3: define product terms using variables common to all minter's in group
  Step 4: sum all essential groups plus a minimal set of remaining groups to obtain a minimum SOP.

K-maps







K-maps

B \ A	0	1
0	0	2
1	1	3

C \ AB	00	01	11	10
0	0	2	6	4
1	1	3	7	5

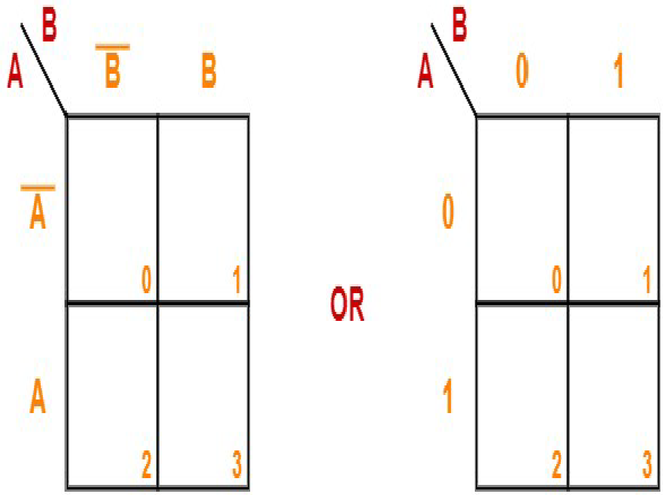
CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Two Variables K-Map

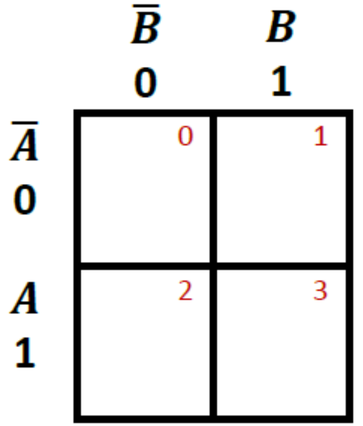
Number of input variables are 2

Hence the number of squares = $2^n = 2^2 = 4$

Inputs A	Inputs B	Decimal equivalent	Minterms		Output F
0	0	0	m_0	$\bar{A}\bar{B}$	
0	1	1	m_1	$\bar{A}B$	
1	0	2	m_2	$A\bar{B}$	
1	1	3	m_3	AB	



Two Variable K Map



Example

A	B	X	
0	0	1	$\overline{A}\,\overline{B}$
0	1	0	
1	0	0	
1	1	1	AB

$$X = \overline{A}\,\overline{B} + AB$$

	$\overline{B}$	B
$\overline{A}$	1	0
A	0	1

Example

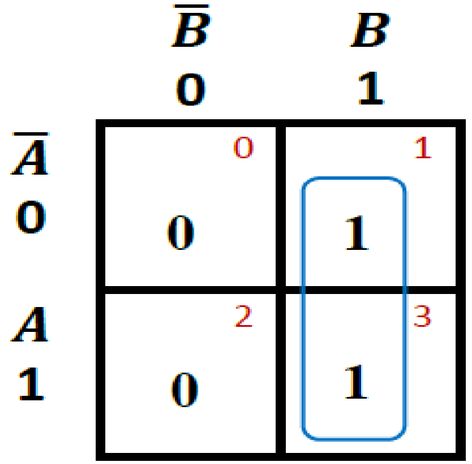
- simplify the Boolean expression by using K-Maps

$$F = \bar{A}B + AB$$

Solution:

Number of input variables are 2

Hence the number of squares $= 2^n = 2^2 = 4$



$$F = B$$

Example

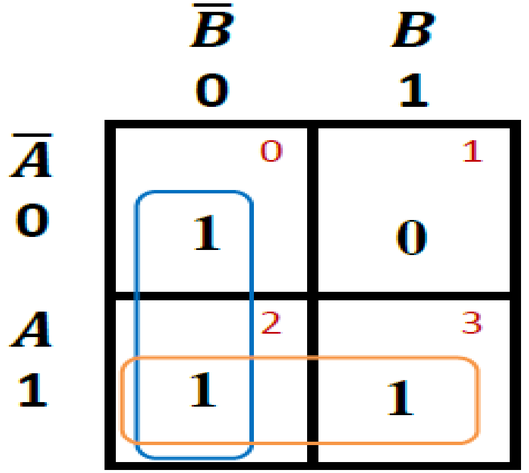
Example: simplify the Boolean expression by using K-Map

$$F(A, B) = \sum m(2, 0, 3)$$

Solution:

Number of input variables are 2

Hence the number of squares $= 2^n = 2^2 = 4$



$$F(A, B) = \overline{B} + A$$

Example

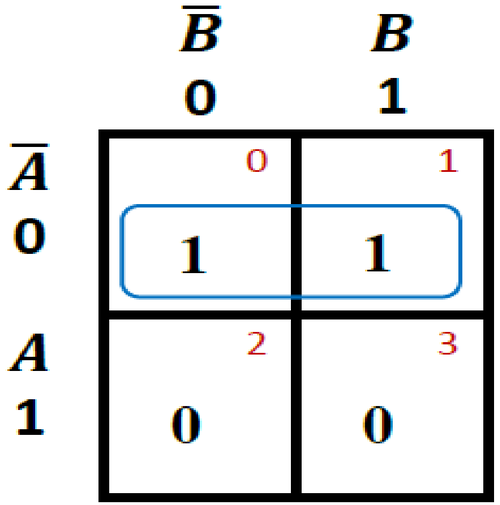
Example: simplify the Boolean expression by using K-Map

$$F = \bar{A}B + \bar{A}\bar{B}$$

Solution:

Number of input variables are 2

Hence the number of squares $= 2^n = 2^2 = 4$



$$F = \bar{A}$$

Example

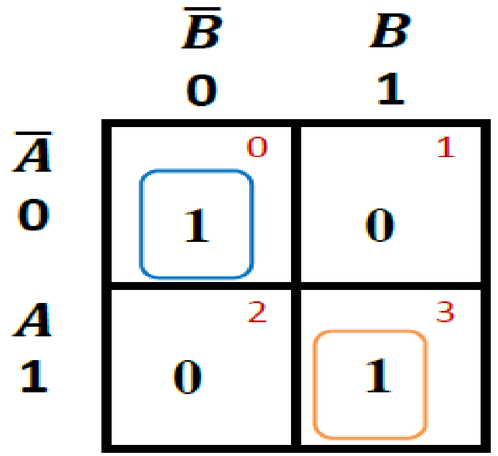
Example: simplify the Boolean expression by using K-Map

$$F(A,B) = \sum m(0,3)$$

Solution:

Number of input variables are 2

Hence the number of squares $= 2^n = 2^2 = 4$



$$F(A,B) = \sum m(0,3) = \overline{A}\overline{B} + AB$$

Three Variables K-Map

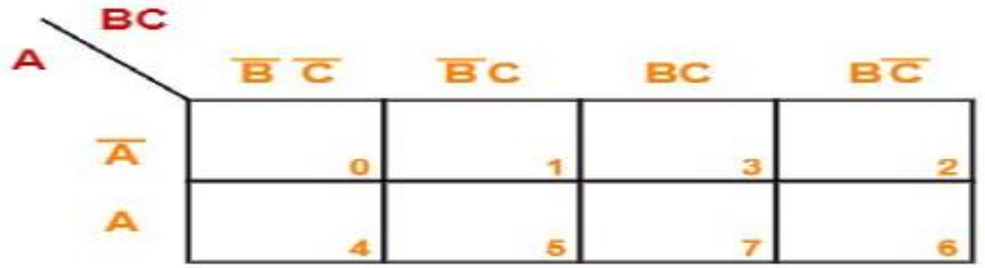
Number of input variables are 3

Hence the number of squares = $2^n = 2^3 = 8$

The truth table is

Inputs A	B	C	Decimal equivalent	Minterms		Output F
0	0	0	0	m_0	$\bar{A}\bar{B}\bar{C}$	
0	0	1	1	m_1	$\bar{A}\bar{B}C$	
0	1	0	2	m_2	$\bar{A}B\bar{C}$	
0	1	1	3	m_3	$\bar{A}BC$	
1	0	0	4	m_4	$A\bar{B}\bar{C}$	
1	0	1	5	m_5	$A\bar{B}C$	
1	1	0	6	m_6	$AB\bar{C}$	
1	1	1	7	m_7	ABC	

	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1	3	2
A 1	4	5	7	6



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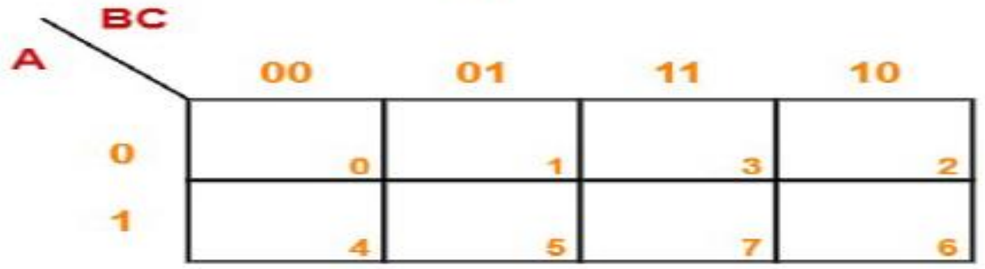
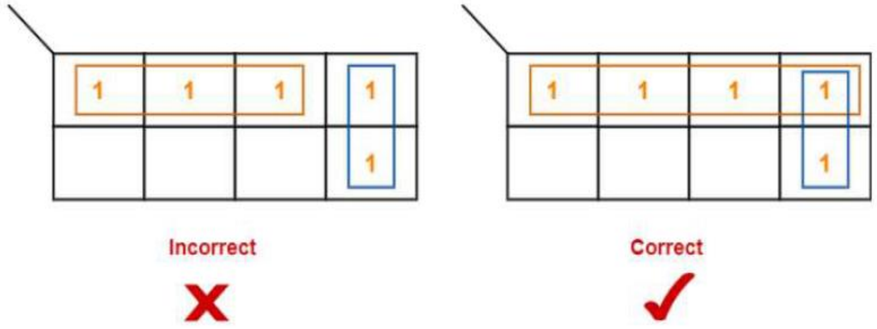


Three Variable K Map

Example-



Example

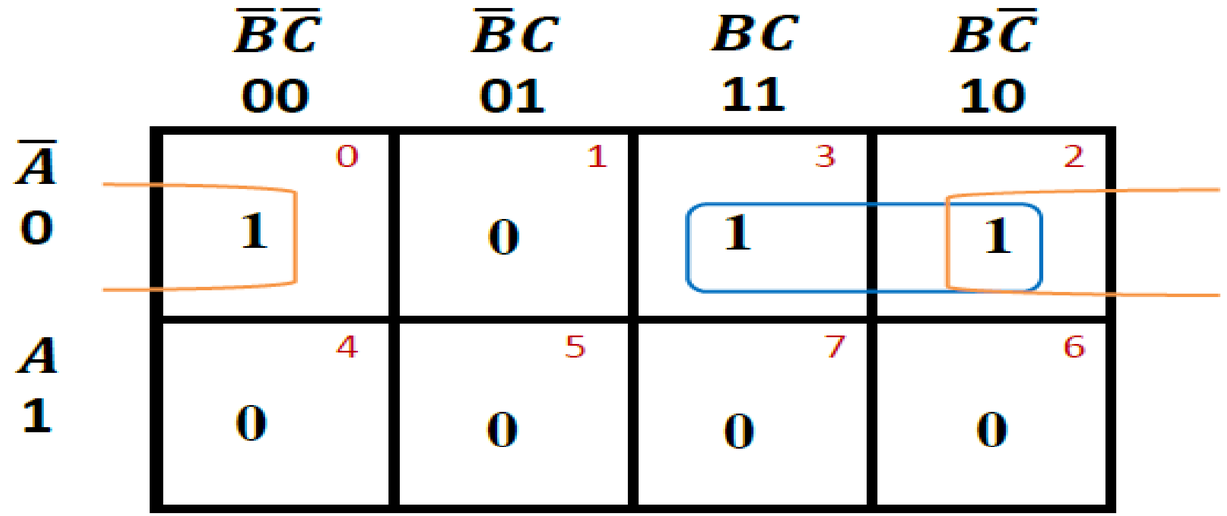
Example: simplify the Boolean expression by using K-Map

$$F(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$$

Solution:

Number of input variables are 3

Hence the number of squares $= 2^n = 2^3 = 8$



$$F(A,B,C) = \bar{A}\bar{C} + \bar{A}B$$

Example

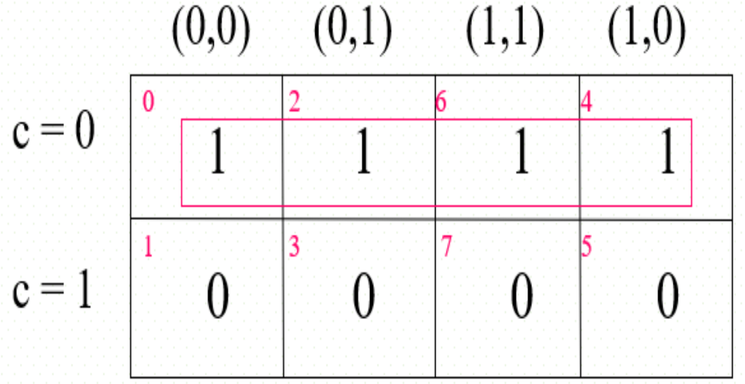
A	B	C	Y
0	0	0	**1**
0	0	1	**1**
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Y	AB 00	01	11	10
C 0	**1**	0	0	0
1	**1**	0	0	0

Y	AB 00	01	11	10
C 0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

Example

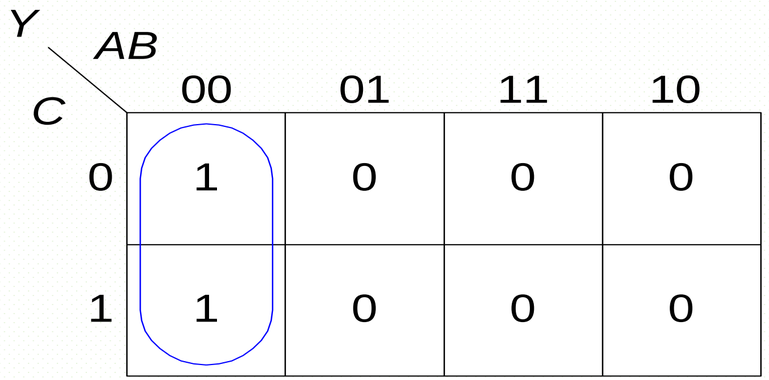
Id	a	b	c	f (a,b,c)
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	0



f(a,b,c) = c’

Example

A	B	C	Y
0	0	0	**1**
0	0	1	**1**
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0



$$y(A,B)=A'B'C'+A'B'C= A'B'(C'+C)=A'B'$$

Example

Example: simplify the Boolean expression by using K-Map

$$F(A,B,C) = \sum m(0,3,7,6)$$

Solution:

Number of input variables are 3

Hence the number of squares $= 2^n = 2^3 = 8$

	$\overline{B}\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$ 0	0: 1	1: 0	3: 1	2: 0
A 1	4: 0	5: 0	7: 1	6: 1

$$F(A,B,C) = \overline{A}\overline{B}\overline{C} + BC + AB$$

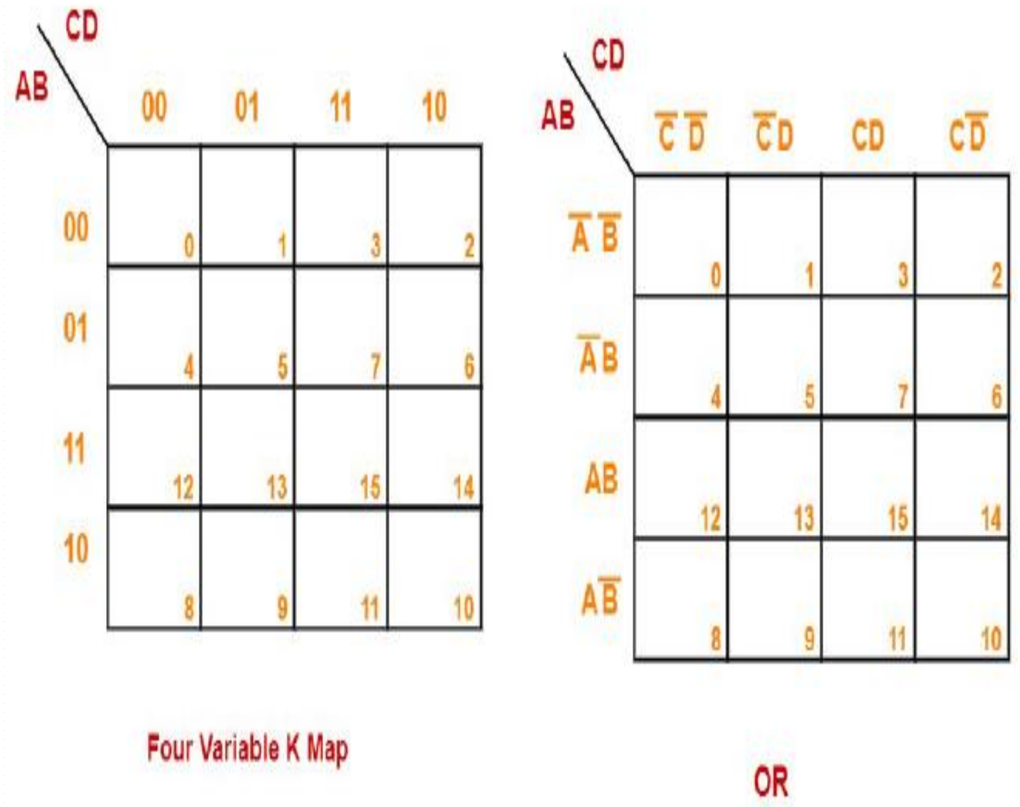
Four Variables K-map

Number of input variables are 4

Hence the number of squares $= 2^n = 2^4 = 16$

The truth table is

Inputs A	B	C	D	Decimal equivalent	Minterms		Output F
0	0	0	0	0	m_0	$\bar{A}\bar{B}\bar{C}\bar{D}$	
0	0	0	1	1	m_1	$\bar{A}\bar{B}\bar{C}D$	
0	0	1	0	2	m_2	$\bar{A}\bar{B}C\bar{D}$	
0	0	1	1	3	m_3	$\bar{A}\bar{B}CD$	
0	1	0	0	4	m_4	$\bar{A}B\bar{C}\bar{D}$	
0	1	0	1	5	m_5	$\bar{A}B\bar{C}D$	
0	1	1	0	6	m_6	$\bar{A}BC\bar{D}$	
0	1	1	1	7	m_7	$\bar{A}BCD$	
1	0	0	0	8	m_8	$A\bar{B}\bar{C}\bar{D}$	
1	0	0	1	9	m_9	$A\bar{B}\bar{C}D$	
1	0	1	0	10	m_{10}	$A\bar{B}C\bar{D}$	
1	0	1	1	11	m_{11}	$A\bar{B}CD$	
1	1	0	0	12	m_{12}	$AB\bar{C}\bar{D}$	
1	1	0	1	13	m_{13}	$AB\bar{C}D$	
1	1	1	0	14	m_{14}	$ABC\bar{D}$	
1	1	1	1	15	m_{15}	$ABCD$	



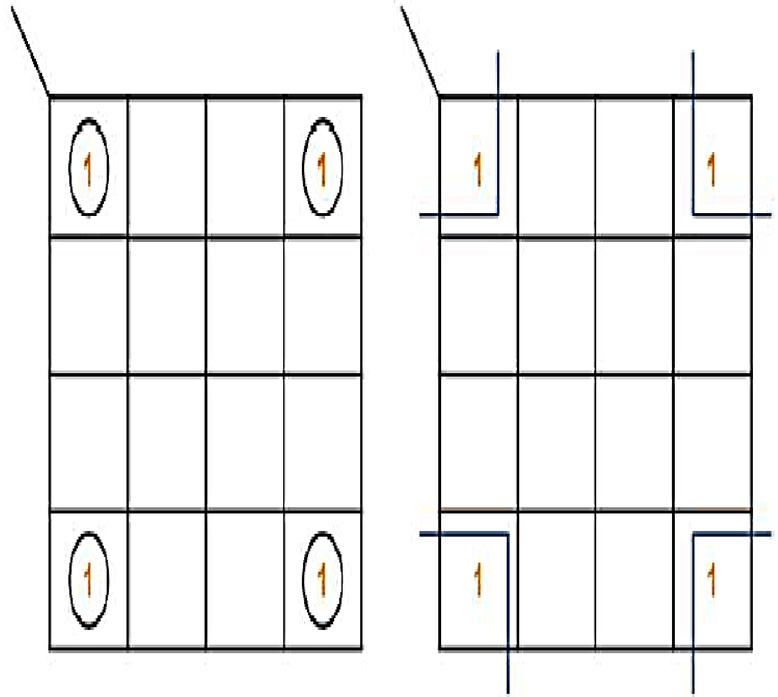
Four Variable K Map

OR

Four Variables K-map

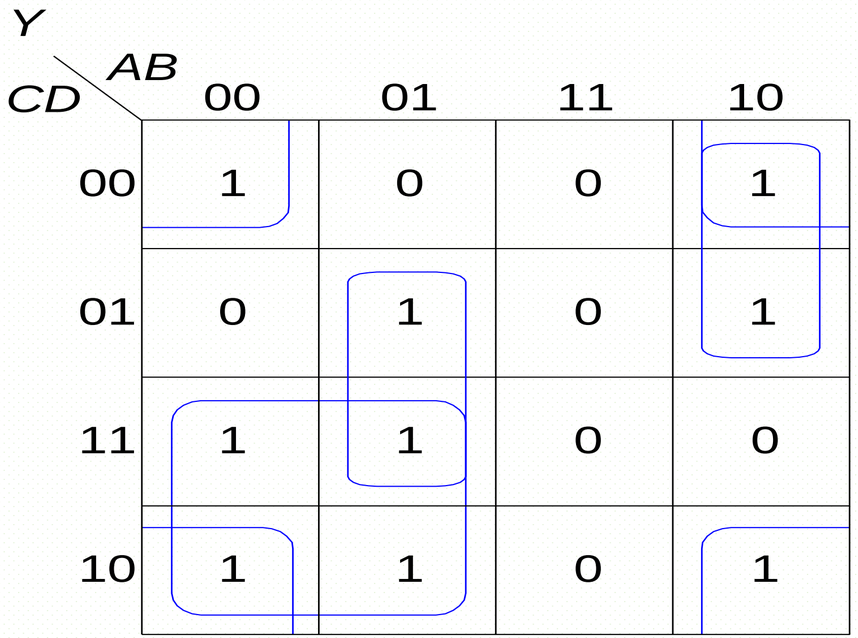
AB \ CD	00	01	11	10
00	$\overline{A}\overline{B}\overline{C}\overline{D}$	$\overline{A}\overline{B}\overline{C}D$	$\overline{A}\overline{B}CD$	$\overline{A}\overline{B}C\overline{D}$
01	$\overline{A}B\overline{C}\overline{D}$	$\overline{A}B\overline{C}D$	$\overline{A}BCD$	$\overline{A}BC\overline{D}$
11	$AB\overline{C}\overline{D}$	$AB\overline{C}D$	$ABCD$	$ABC\overline{D}$
10	$A\overline{B}\overline{C}\overline{D}$	$A\overline{B}\overline{C}D$	$A\overline{B}CD$	$A\overline{B}C\overline{D}$

Example-



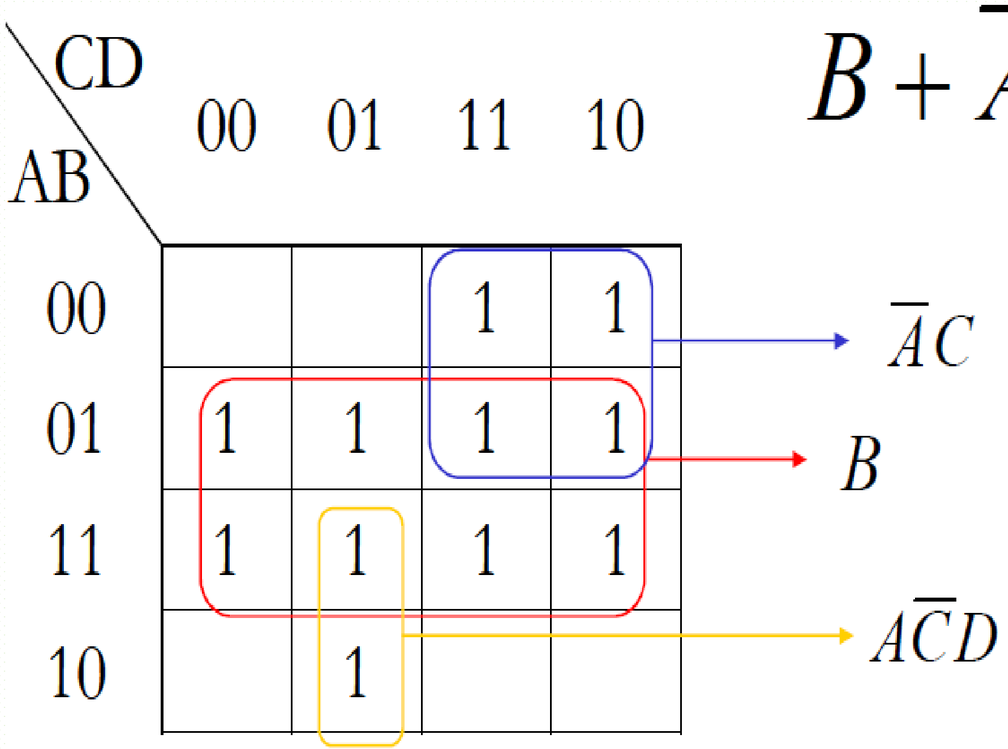
Example

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0



$$Y = \bar{A}C + \bar{A}BD + A\bar{B}\bar{C} + \bar{B}\bar{D}$$

Example



$$B + \overline{A}C + A\overline{C}D$$

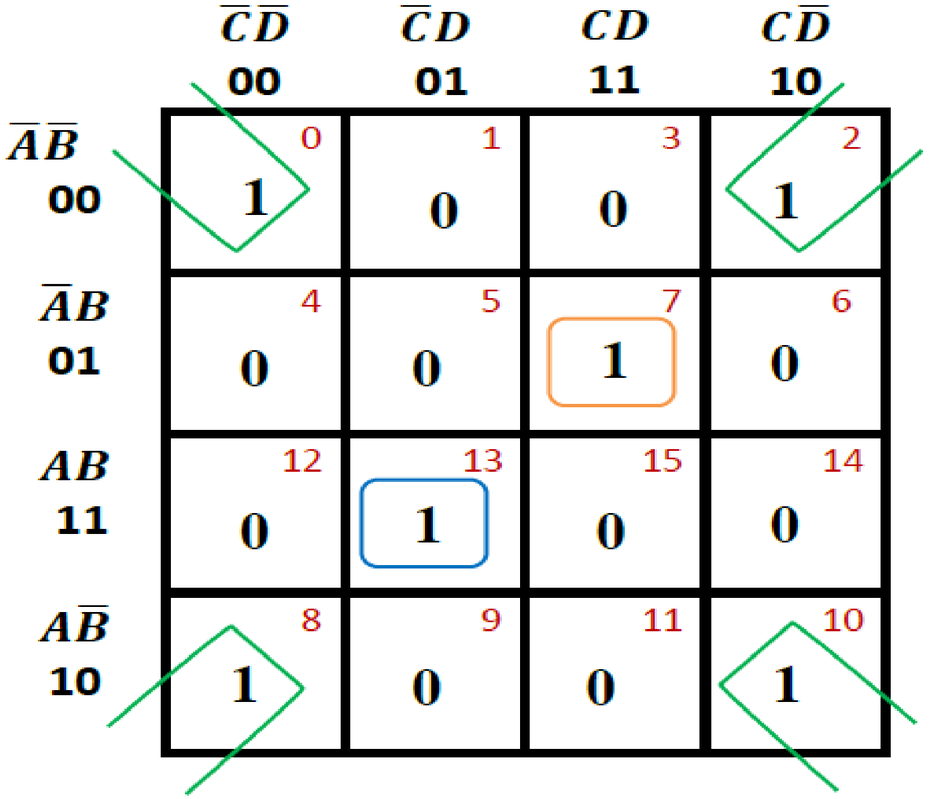
Example

Example: simplify the Boolean expression by using K-Map

$$F(A,B,C,D) = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{A}BCD + AB\bar{C}D$$

Solution: Number of input variables are 4

Hence the number of squares = $2^n = 2^4 = 16$



$$F(A,B,C,D) = \bar{B}\bar{D} + \bar{A}BCD + AB\bar{C}D$$

Example

Example: simplify the Boolean expression by using K-Map

$$F(A,B,C,D) = \sum m(0,2,4,6,12,14,15,8,10)$$

Solution: Number of input variables are 4

Hence the number of squares = $2^n = 2^4 = 16$

	$\overline{C}\overline{D}$ 00	$\overline{C}D$ 01	CD 11	$C\overline{D}$ 10
$\overline{A}\overline{B}$ 00	0: **1**	1: **0**	3: **0**	2: **1**
$\overline{A}B$ 01	4: **1**	5: **0**	7: **0**	6: **1**
AB 11	12: **1**	13: **0**	15: **1**	14: **1**
$A\overline{B}$ 10	8: **1**	9: **0**	11: **0**	10: **1**

$$F(A,B,C,D) = \overline{D} + ABC$$

Example

A	B	C	D	X	
0	0	0	0	0	
0	0	0	1	1	$\overline{A}\overline{B}\overline{C}D$
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	1	$\overline{A}B\overline{C}D$
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	1	$AB\overline{C}D$
1	1	1	0	0	
1	1	1	1	1	$ABCD$

	$\overline{C}\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\overline{B}$	0	1	0	0
$\overline{A}B$	0	1	0	0
AB	0	1	1	0
$A\overline{B}$	0	0	0	0

$$X = \overline{A}B\overline{C}D + \overline{A}B\overline{C}D + AB\overline{C}D + ABCD$$

Example

Example: simplify the Boolean expression by using K-Map

$$F(A,B,C,D) = \sum m(3,7,9,11) + \sum d(1,5,12,14)$$

Solution: Number of input variables are 4

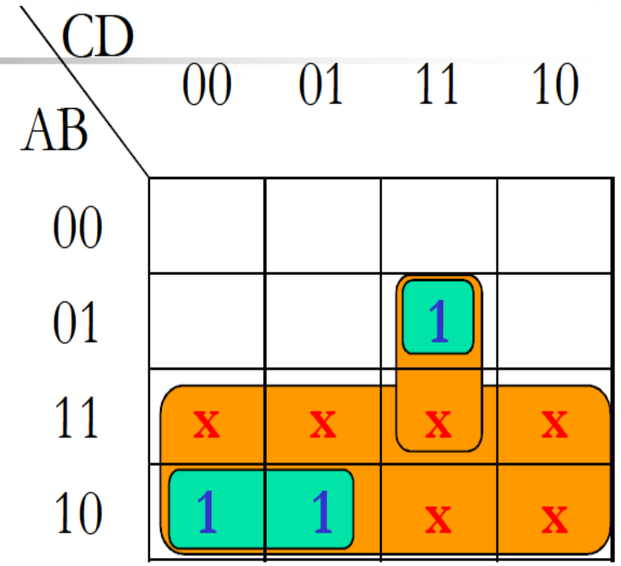
Hence the number of squares $= 2^n = 2^4 = 16$

	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0 (0)	X (1)	1 (3)	0 (2)
$\bar{A}B$ 01	0 (4)	X (5)	1 (7)	0 (6)
AB 11	X (12)	0 (13)	0 (15)	X (14)
$A\bar{B}$ 10	0 (8)	1 (9)	1 (11)	0 (10)

$$F(A,B,C,D) = \bar{A}D + \bar{B}D$$

Example

INPUTS				O/P
A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X



Without "don't care"

$$Y = A\overline{B}\,\overline{C} + \overline{A}BCD$$

With "don't care"

$$Y = A + BCD$$

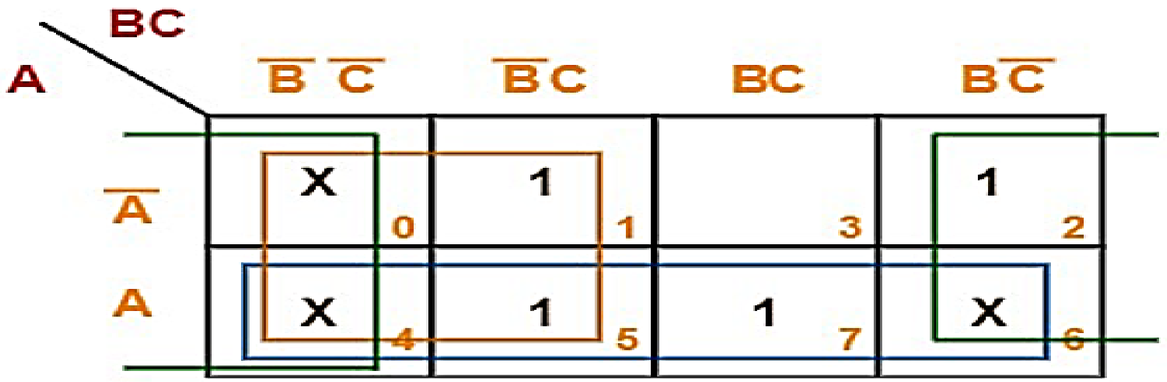
Example

$$F(A, B, C) = \Sigma m(1, 2, 5, 7) + \Sigma d(0, 4, 6)$$

Solution-

- Since the given boolean expression has 3 variables, so we draw a 2 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-



Now,

F(A, B, C)

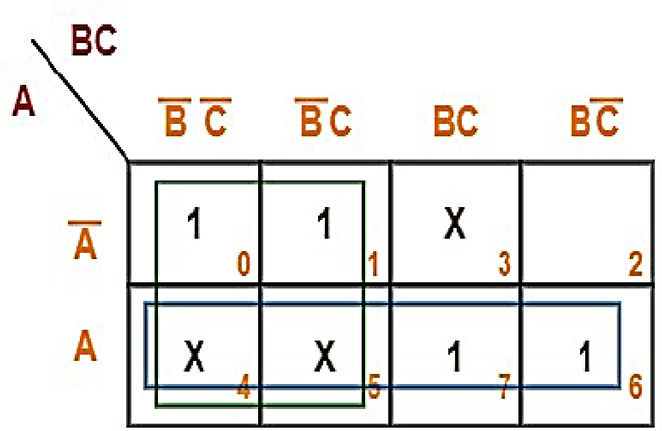
= (A + A')(B'C' + B'C) + A(B'C' + B'C + BC + BC') + (A + A')(B'C' + BC')

= B' + A + C'

Thus, minimized boolean expression is-

F(A, B, C) = A + B' + C'

Example



Now,

F(A, B, C)

= (A + A')(B'C' + B'C) + A(B'C' + B'C + BC + BC')

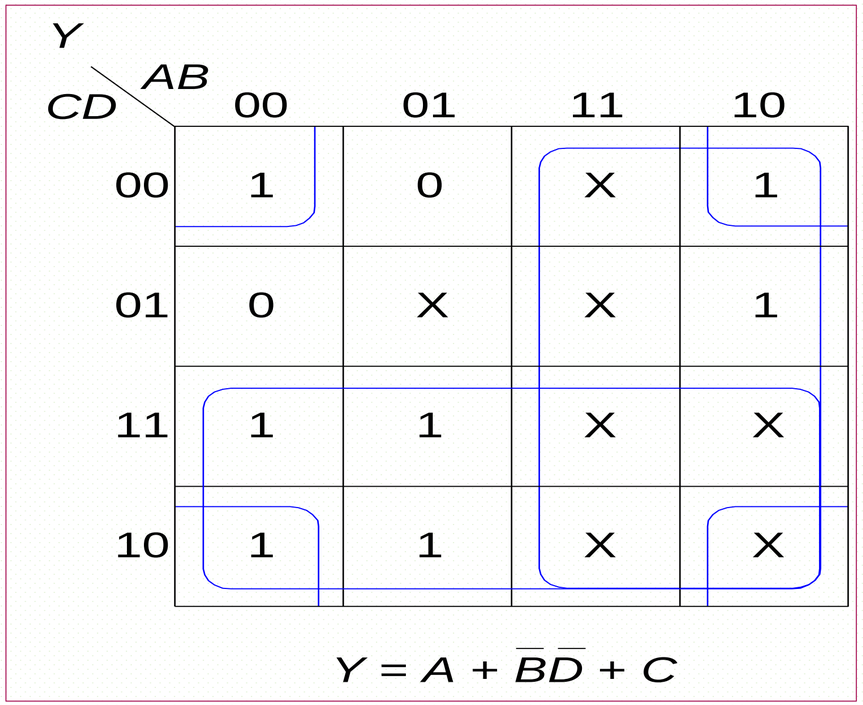
= B' + A

Thus, minimized boolean expression is-

F(A, B, C) = A + B'

Example

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X



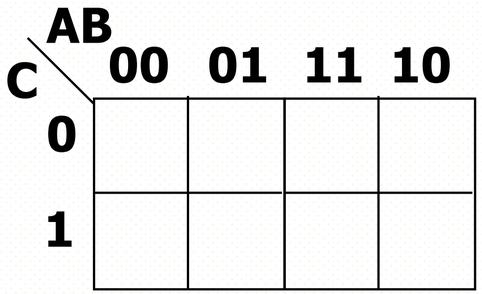
Example Homework

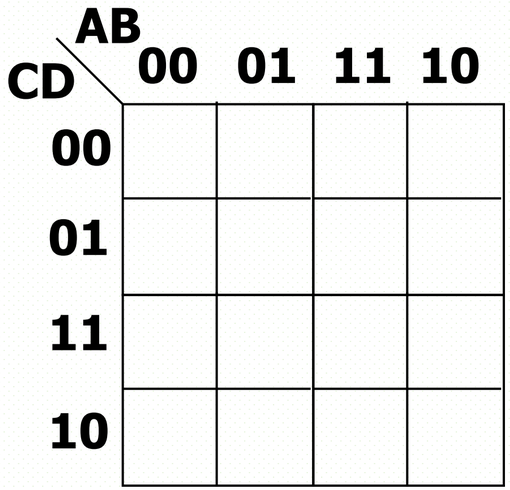
Id	a	b	c	f (a,b,c,d)
0	0	0	0	1
1	0	0	1	1
2	0	1	0	-
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

Example Homework

$F(A,B,C) = \Sigma m(0,3,6,7)$

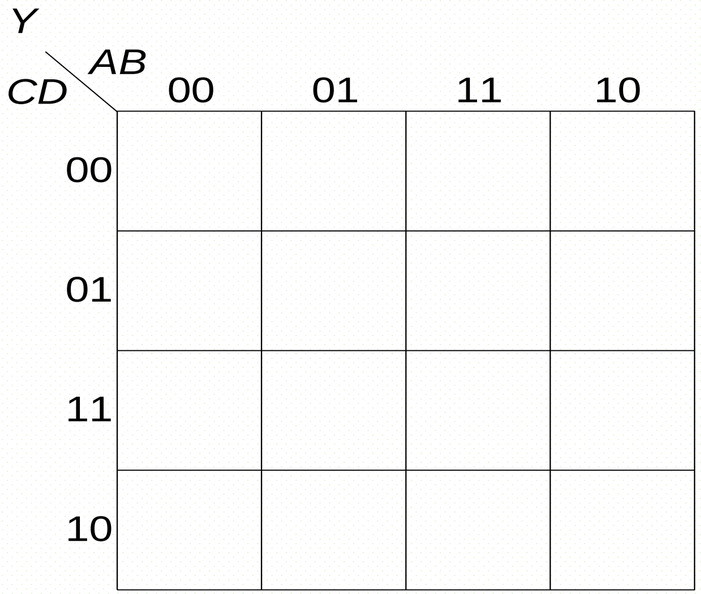
$F(A,B,C,D) = \Sigma m(0,3,7,8,11,15)$





Example Homework

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X



Example Homework

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y	AB 00	01	11	10
CD 00	1	0	X	1
01	0	X	X	1
11	1	1	X	X
10	1	1	X	X