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# LECTURE 4: Moment and Couple

## 1. Introduction:

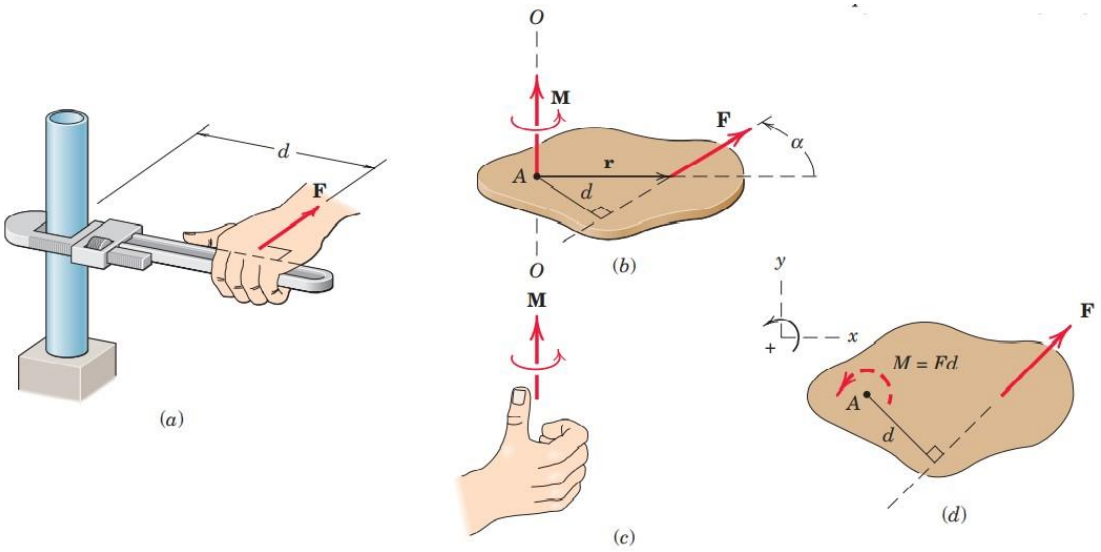
In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line that neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the moment **M** of the force. Moment is also referred to as torque.

## 2. Moment about a Point:

Figure below shows a two-dimensional body acted on by a force **F** in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis OO perpendicular to the plane of the body is proportional both to the magnitude of the force and to the moment arm d, which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as:

$$M = Fd$$

The basic units of moment in SI units are newton-meters (N.m) and in the U.S. customary system are pound-feet (lb-ft).



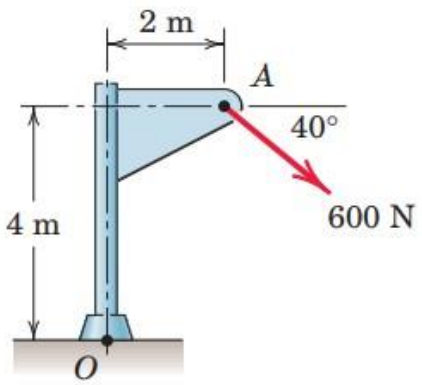

Moment directions may be accounted for by using a stated sign convention, such as a plus sign (+) for counterclockwise moments and a minus sign (-) for clockwise moments, or vice versa. Sign consistency within a given problem is essential.

## 3. Varignon's Theorem:

One of the most useful principles of mechanics is Varignon's theorem, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

**Example 1:**

Calculate the magnitude of the moment about the base point O of the 600-N force in two different ways.

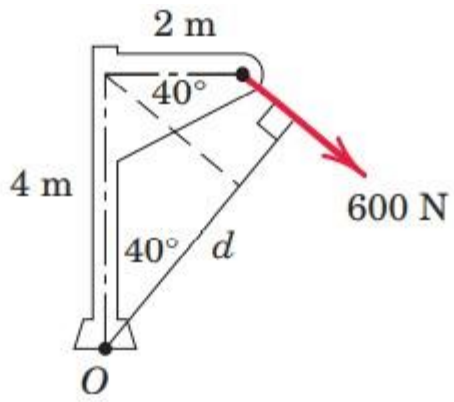


**Solution.** *(I)* The moment arm to the 600-N force is

$$d = 4 \cos 40° + 2 \sin 40° = 4.35 \text{ m}$$

1 By $M = Fd$ the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N} \cdot \text{m} \qquad \textit{Ans.}$$

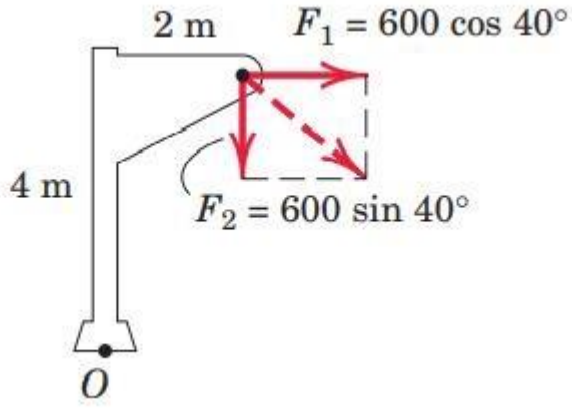



*(II)* Replace the force by its rectangular components at $A$,

$$F_1 = 600 \cos 40° = 460 \text{ N}, \qquad F_2 = 600 \sin 40° = 386 \text{ N}$$

By Varignon's theorem, the moment becomes

2 $$M_O = 460(4) + 386(2) = 2610 \text{ N} \cdot \text{m} \qquad \textit{Ans.}$$



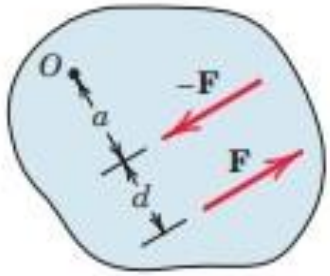
## 4. Couple:

The moment produced by two equal, opposite, and noncollinear forces is called a couple. Couples have certain unique properties and have important applications in mechanics. Consider the action of two equal and opposite forces F and -F a distance d apart, as shown in Fig. below.. These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as O in their plane is the couple M. This couple has a magnitude of:

$$M = F(a + d) - Fa$$

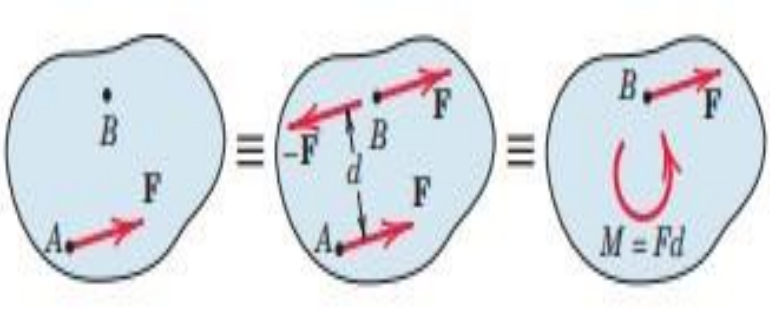
or

$$M = Fd$$




## 5. Force–Couple Systems

The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force, and to rotate the body about any fixed axis which does not intersect the line of the force. We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

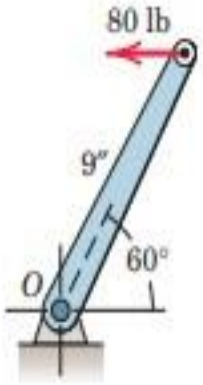



The original force at A is replaced by the same force acting at a different point B and a couple, without altering the external effects of the original force on the body. The combination of the force and couple in the right-hand part of Fig. above is referred to as a force–couple system.

By reversing this process, we can combine a given couple and a force which lies in the plane of the couple (normal to the couple vector) to produce a single, equivalent force. Replacement of a force by an equivalent force–couple system, and the reverse procedure, have many applications in mechanics and should be mastered.

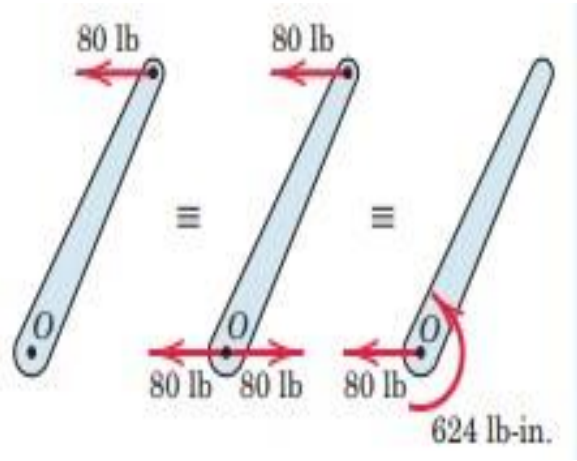
**Illustrative example:**

Replace the horizontal 80-lb force acting on the lever by an equivalent system consisting of a force at O and a couple.



**Solution:**

We apply two equal and opposite 80-lb forces at O and identify the counterclockwise couple:

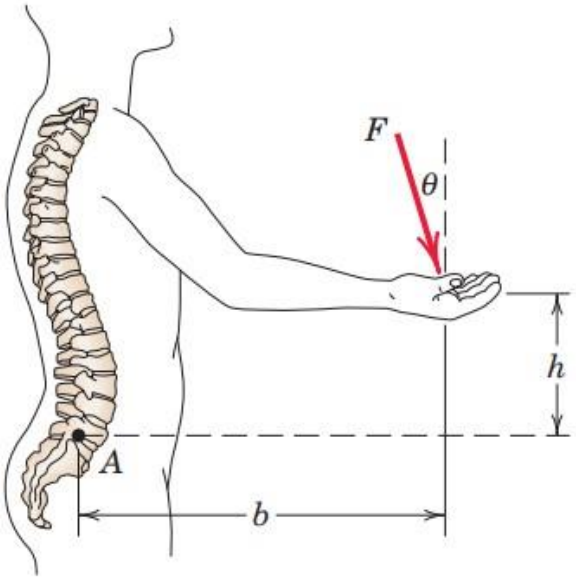



$$M = Fd = 80(9 \sin 60^{o}) = 624 \text{ lb-in}$$

Thus, the original force is equivalent to the 80-lb force at O and the 624-lb-in. couple as shown in the third of the three equivalent figures above.

**H.W.1**

The lower lumbar region $A$ of the spine is the part of the spinal column most susceptible to abuse while resisting excessive bending caused by the moment about $A$ of a force $F$. For given values of $F$, $b$, and $h$, determine the angle $\theta$ which causes the most severe bending strain.




**H.W.2:**

Compute the moment of the 0.4-lb force about the pivot $O$ of the wall-switch toggle.

