
LECTURE 2: Force Systems

1. Introduction

Before dealing with a group or system of forces, it is necessary to examine the properties of a single force in some detail. A force has been defined in the previous lecture as *an action of one body on another*. A force is a vector quantity because its effect depends on the direction as well as on the magnitude of the action. Thus, forces may be combined according to the parallelogram law of vector addition.

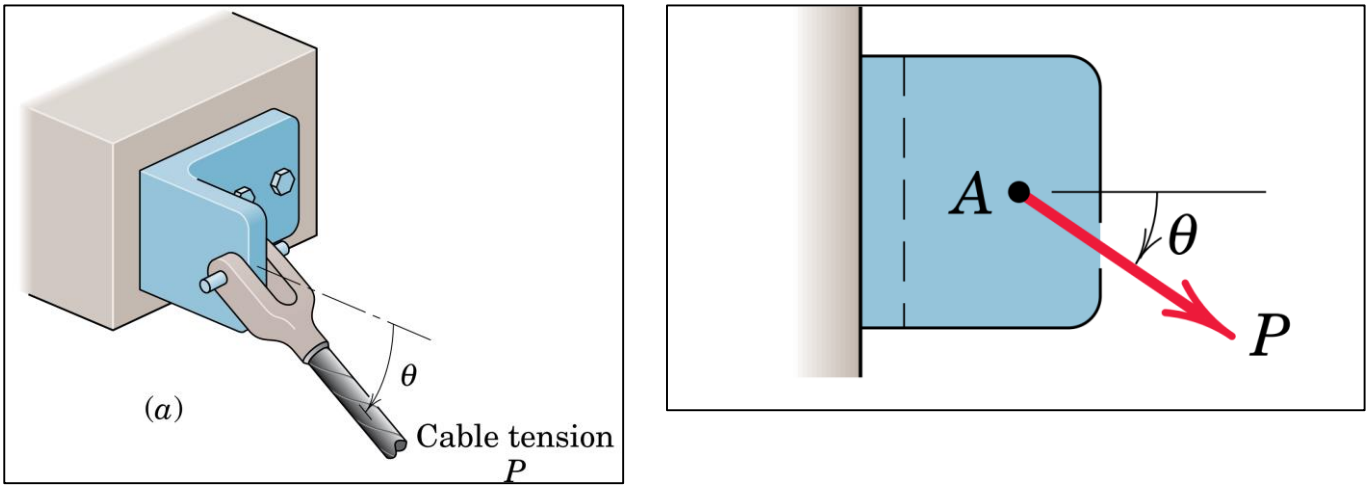


Fig. 1

In Fig 1, the effect of this action on the bracket depends on P, the angle θ , and the location of the point of application A. Changing any one of these three specifications will alter the effect on the bracket, such as the force in one of the bolts which secure the bracket to the base, or the internal force and deformation in the material of the bracket at any point.

Thus, the complete specification of the action of a force must include its *magnitude, direction, and point of application*, and therefore we must treat it as a fixed vector.

2. Two-Dimensional Force Systems

The most common two-dimensional resolution of a force vector is into rectangular components. It follows from the parallelogram rule that the vector $\boldsymbol{F}$ of Fig. 2 may be written as

$$\boldsymbol{F} = \boldsymbol{F}_x + \boldsymbol{F}_y \qquad (1)$$

where $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$ are vector components of $\boldsymbol{F}$ in the x- and y-directions. Each of the two vector components may be written as a scalar times the appropriate unit vector (**i**. and **j).** Thus, we may write:

$$\boldsymbol{F} = F_x\boldsymbol{i} + \boldsymbol{F}_y\boldsymbol{j} \qquad (1)$$

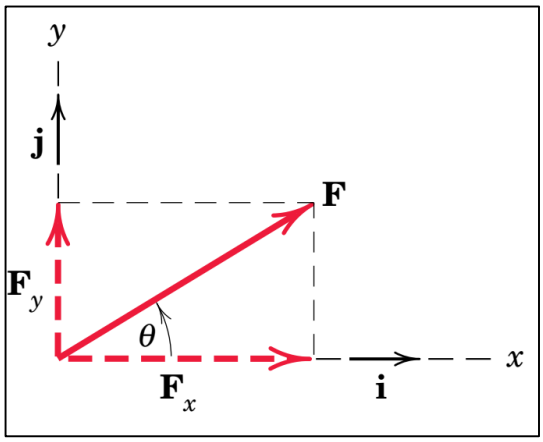


Fig. 2

The scalar components can be positive or negative, depending on the quadrant into which F points. For the force vector of Fig. 2, the x and y scalar components are both positive and are related to the magnitude and direction of F by

$$F_x = F\cos\theta \qquad (2)$$

$$F_y = F\sin\theta \qquad (3)$$

$$F = \sqrt{F_x^2 + F_y^2} \qquad (4)$$

$$\theta = \tan^{-1}\frac{F_y}{F_x} \qquad (5)$$

Example 1:

The forces $F1$, $F2$, and $F3$ in Fig. 3, all of which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components of each of the three forces.

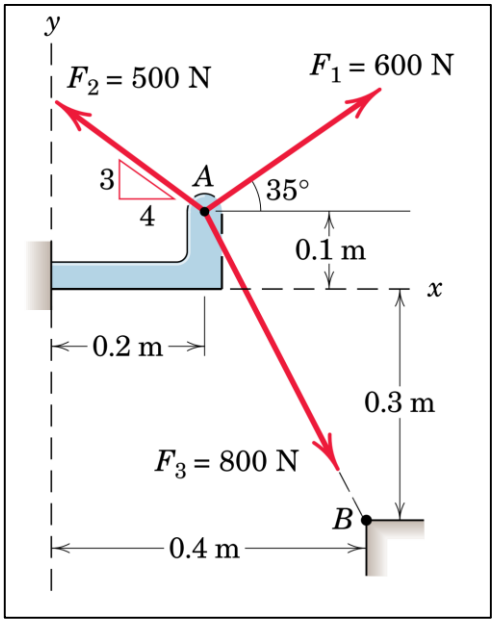


Fig. 3

Solution.

The scalar components of $F1$, from Fig. 4.a, are

$$F1_x = 600\, cos35^0 = 491N$$

$$F1_y = 600\, sin35^0 = 344N$$

The scalar components of F2, from Fig. 4.b, are

$$F2_x = -500 * \frac{4}{5} = -400N$$

$$F2_y = 500 * \frac{3}{5} = 300N$$

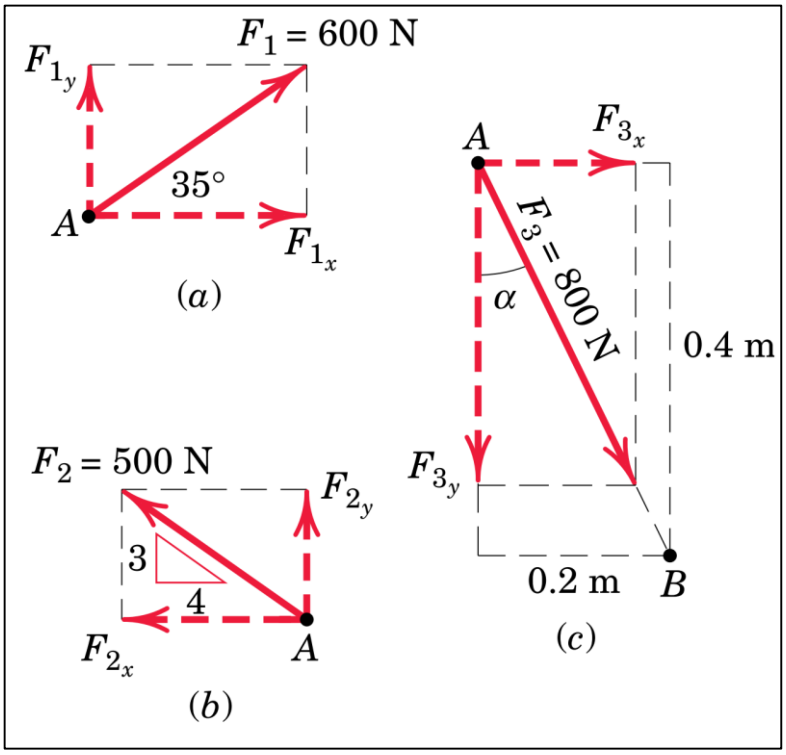


Fig. 4

Note that the angle which orients $F2$ to the x-axis is never calculated. The cosine and sine of the angle are available by inspection of the 3-4-5 triangle. Also note that the x scalar component of $F2$ is negative by inspection.

The scalar components of $F3$ can be obtained by first computing the angle of Fig. 4.c.

$$\alpha = tan^{-1}\frac{0.2}{0.4} = 26.6^0$$

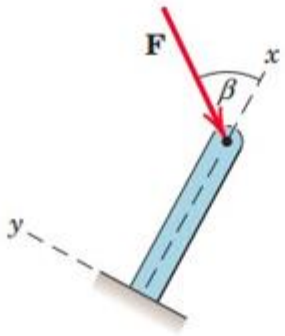
Then,

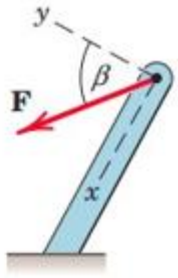
$$F3_x = F3\, sin\alpha = 800\, sin26.6^0 = 358N$$

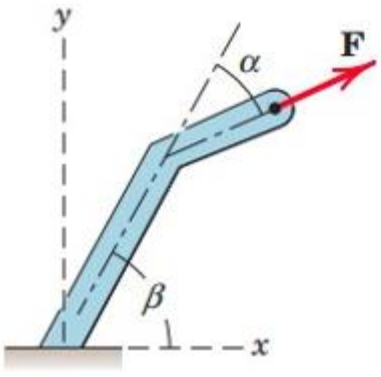
$$F3_y = -F3\, cos\alpha = -800 cos26.6^0 = -716N$$

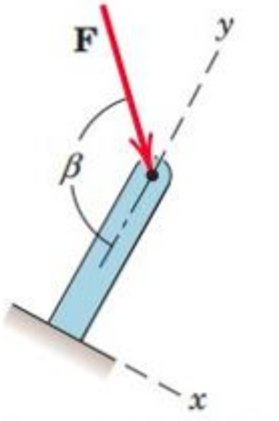
H.W.:

Find the x and y scalar components of the force F for each case shown below:









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Reference: Engineering Mechanics, Statics, J. L. Meriam and L. G. Kraige, John Wiley & Sons, Inc