Dept. of Medical Instrumentations Engineering

First Stage: **Academic Year: 2020-2021**

Subject: Mechanics **Instructor: Assist. Prof. Dr. Saad Alkhfaji**

==========

LECTURE 5:

Equilibrium of Force System

Introduction:

In this lecture, we will apply our knowledge of the properties of forces, moments, and couples studied in the previous lectures to solve problems involving rigid bodies in equilibrium. When a body is in equilibrium, the resultant of all forces acting on it is zero. Thus, the resultant force **R** and the resultant couple **M** are both zero.

The equilibrium equations (conditions for equilibrium) are:

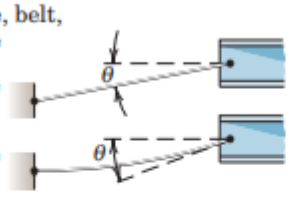
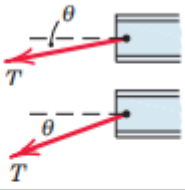
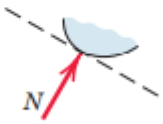
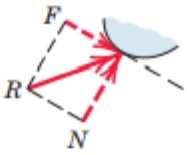
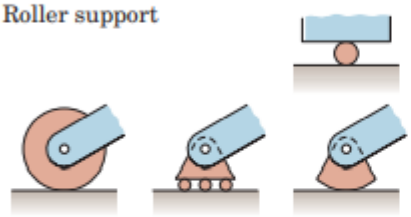
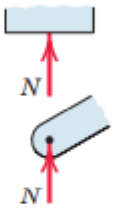
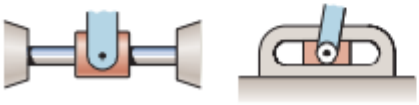
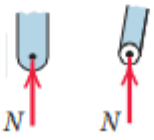
$$\mathbf{R} = \Sigma \mathbf{F} = \mathbf{0} \qquad \mathbf{M} = \Sigma \mathbf{M} = \mathbf{0}$$

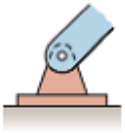
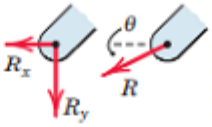
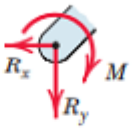
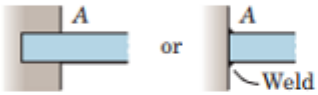
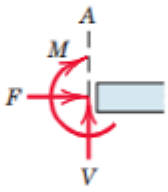
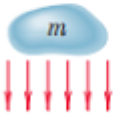
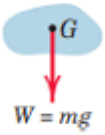
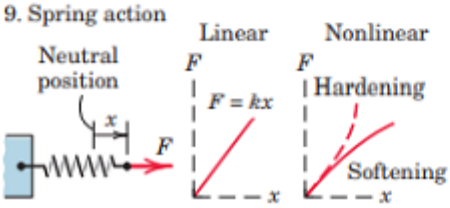
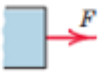
EQUILIBRIUM IN TWO DIMENSIONS: System Isolation and the Free-Body Diagram

Free-body diagram: it is a diagrammatic representation of the isolated system treated as a single body. The diagram shows all forces applied to the system by mechanical contact with other bodies, which are imagined to be removed. The free-body diagram is the most important single step in the solution of problems in mechanics.

Modeling the Action of Forces

The Figure below shows the common types of force application on mechanical systems for analysis in two dimensions. Each example shows the force exerted on the body to be isolated, by the body to be removed. Newton's third law, which notes the existence of an equal and opposite reaction to every action, must be carefully observed. The force exerted on the body in question by a contacting or supporting member is always in the sense to oppose the movement of the isolated body which would occur if the contacting or supporting body were removed.

Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.

Type of Contact and Force Origin	Action on Body to Be Isolated
6. Pin connection 	Pin free to turn  Pin not free to turn  A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 	 Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

Construction of Free-Body Diagrams

The full procedure for drawing a free-body diagram which isolates a body or system consists of the following steps:

Step 1. Decide which system to isolate. The system chosen should usually involve one or more of the desired unknown quantities.

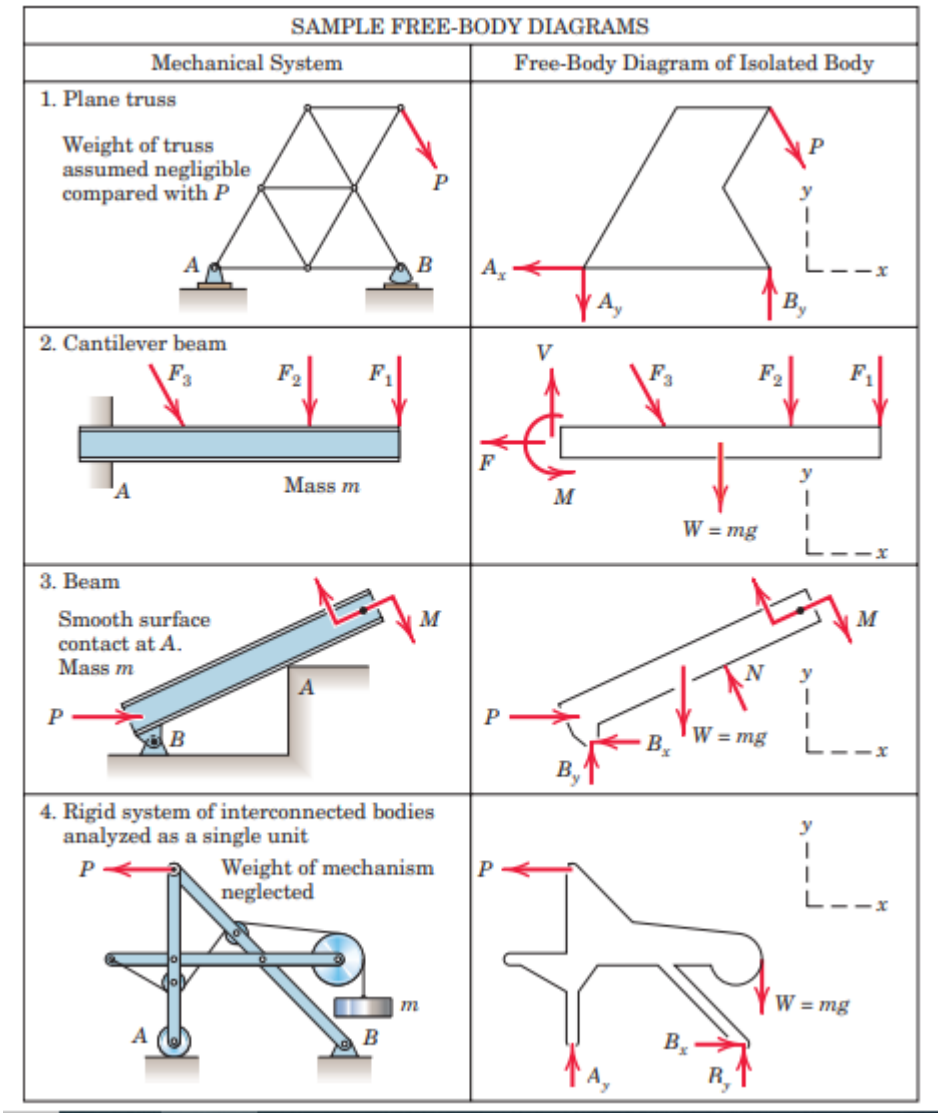
Step 2. Next isolate the chosen system by drawing a diagram which represents its complete external boundary. This boundary defines the isolation of the system from all other attracting or contacting bodies, which are considered removed. This step is often the most crucial of all. Make certain that you have completely isolated the system before proceeding with the next step.

Step 3. Identify all forces which act on the isolated system as applied by the removed contacting and attracting bodies, and represent them in their proper positions on the diagram of the isolated system.

Step 4. Show the choice of coordinate axes directly on the diagram.

Examples of Free-Body Diagrams

Figure in below gives four examples of mechanisms and structures together with their correct free-body diagrams. Dimensions and magnitudes are omitted for clarity. In each case we treat the entire system as a single body, so that the internal forces are not shown.



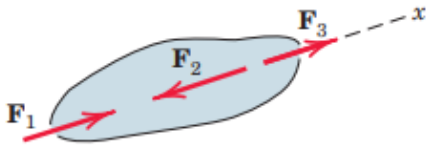
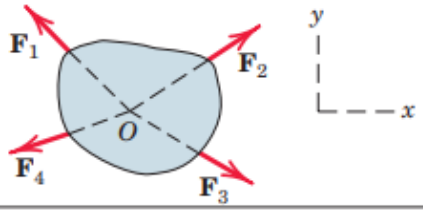
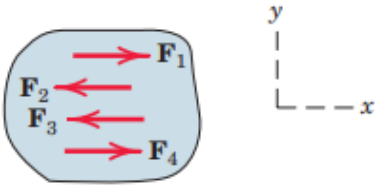
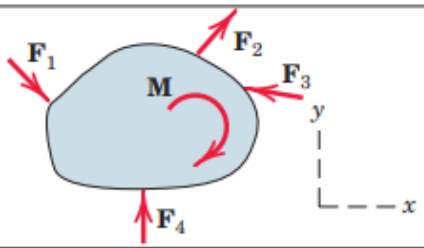
Equilibrium Conditions

Equilibrium can be defined as the condition in which the resultant of all forces and moments acting on a body is zero.

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0$$

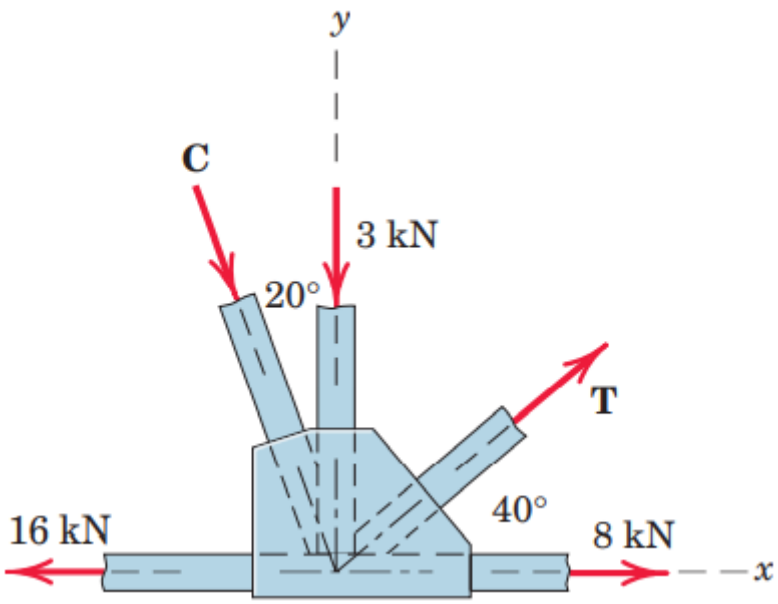
Categories of Equilibrium

Applications of equations above fall naturally into a number of categories which are easily identified. The categories of force systems acting on bodies in two-dimensional equilibrium are summarized in Fig. shown below:

CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

Example 1:

Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint.



Solution:

The given sketch constitutes the free-body diagram of the isolated section of the joint in question and shows the five forces which are in equilibrium

For the x-y axes as shown we have:

$$[\Sigma F_x = 0] \qquad 8 + T\cos 40° + C\sin 20° - 16 = 0$$

$$0.766T + 0.342C = 8 \qquad (a)$$

$$[\Sigma F_y = 0] \qquad T\sin 40° - C\cos 20° - 3 = 0$$

$$0.643T - 0.940C = 3 \qquad (b)$$

Simultaneous solution of Eqs. (a) and (b) produces

$$T = 9.09 \text{ kN} \qquad C = 3.03 \text{ kN} \qquad \textit{Ans.}$$

H.W.

Calculate the tension T in the cable which supports the 1000-lb load with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C.

