Dept. of Medical Instrumentations Engineering

First Stage: **Academic Year: 2020-2021**

Subject: Mechanics **Instructor: Assist. Prof. Dr. Saad Alkhfaji**

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LECTURE 4:

Resultant of a system of forces

Introduction:

The properties of force, moment, and couple were developed in the previous lectures. Now we are ready to describe the resultant action of a group or system of forces. Most problems in mechanics deal with a system of forces, and it is usually necessary to reduce the system to its simplest form to describe its action.

The resultant of a system of forces is the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

Equilibrium of a body is the condition in which the resultant of all forces acting on the body is zero. This condition is studied in statics. When the resultant of all forces on a body is not zero, the acceleration of the body is obtained by equating the force resultant to the product of the mass and acceleration of the body. This condition is studied in dynamics. Thus, the determination of resultants is basic to both statics and dynamics.

The most common type of force system occurs when the forces all act in a single plane, say, the x-y plane, as shown by the system of three forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ in Fig. 1.

The magnitude and direction of the resultant force R can be obtained by forming the force polygon shown in Fig. 2 below, where the forces are added head-to-tail in any sequence.

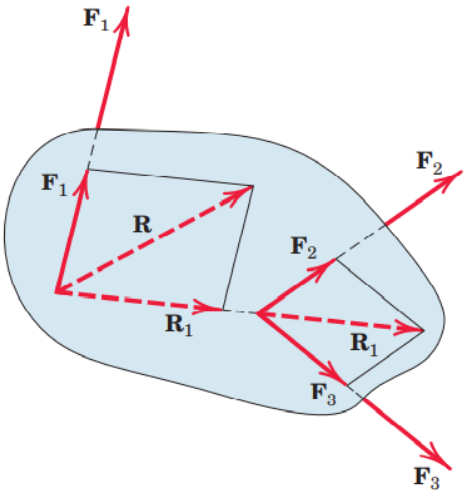


Fig. 1

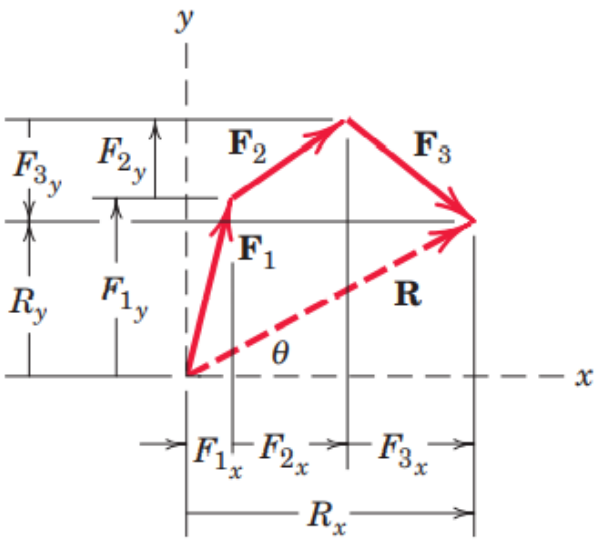


Fig. 2

Thus, for any system of coplanar forces we may write:

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

Principle of Moments:

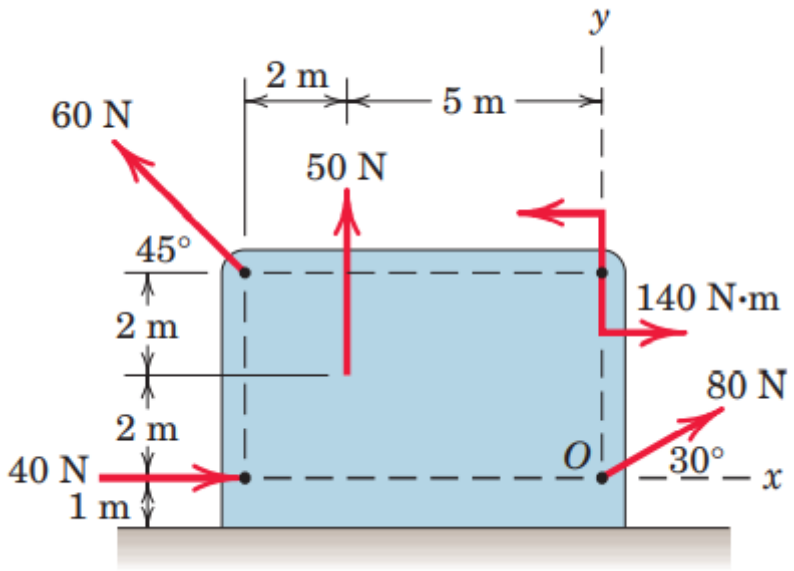
This process is summarized in equation form by:

$$\mathbf{R} = \Sigma \mathbf{F}$$
$$M_O = \Sigma M = \Sigma (Fd)$$
$$Rd = M_O$$

The first two equations above reduce a given system of forces to a force– couple system at an arbitrarily chosen but convenient point O. The last equation specifies the distance d from point O to the line of action of R, and states that the moment of the resultant force about any point O equals the sum of the moments of the original forces of the system about the same point. This extends Varignon's theorem to the case of nonconcurrent force systems; we call this extension the principle of moments.

Example:

Determine the resultant of the four forces and one couple which act on the plate shown below:



Solution:

Point O is selected as a convenient reference point for the force–couple system which is to represent the given system.

$[R_x = \Sigma F_x]$ $\quad R_x = 40 + 80\cos 30° - 60\cos 45° = 66.9$ N

$[R_y = \Sigma F_y]$ $\quad R_y = 50 + 80\sin 30° + 60\cos 45° = 132.4$ N

$[R = \sqrt{R_x^2 + R_y^2}]$ $\quad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3$ N *Ans.*

$\left[\theta = \tan^{-1}\dfrac{R_y}{R_x}\right]$ $\quad \theta = \tan^{-1}\dfrac{132.4}{66.9} = 63.2°$ *Ans.*

$[M_O = \Sigma(Fd)]$ $\quad M_O = 140 - 50(5) + 60\cos 45°(4) - 60\sin 45°(7)$

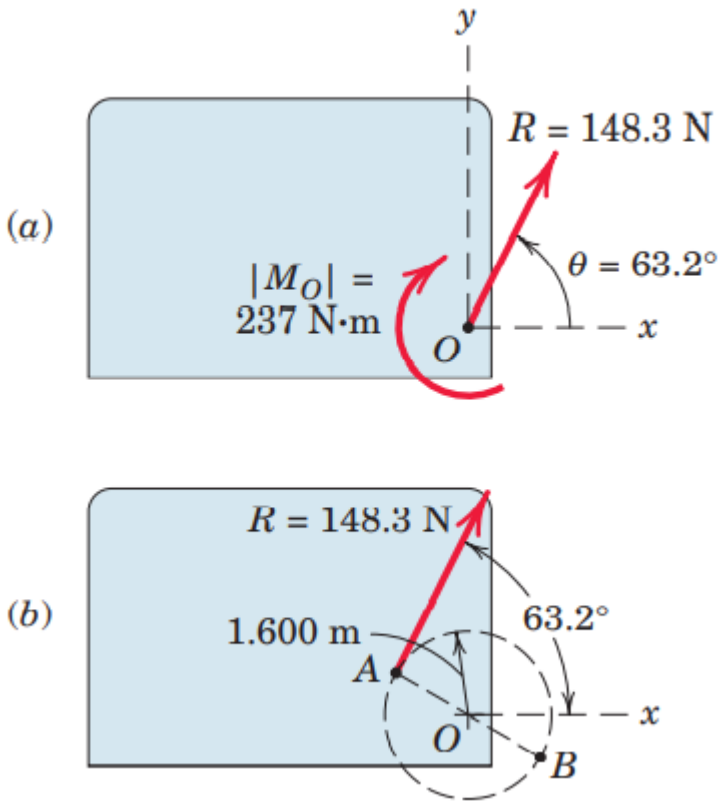
$= -237$ N·m

The force–couple system consisting of **R** and M_O is shown in Fig. *a*.

We now determine the final line of action of **R** such that **R** alone represents the original system.

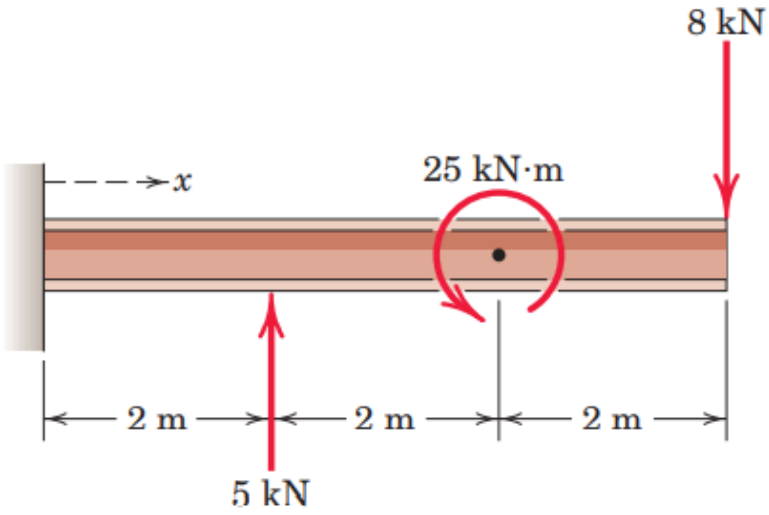
$[Rd = |M_O|]$ $\quad 148.3d = 237$ $\quad d = 1.600$ m *Ans.*

Hence, the resultant **R** may be applied at any point on the line which makes a 63.2° angle with the *x*-axis and is tangent at point *A* to a circle of 1.600-m radius with center *O*, as shown in part *b* of the figure.



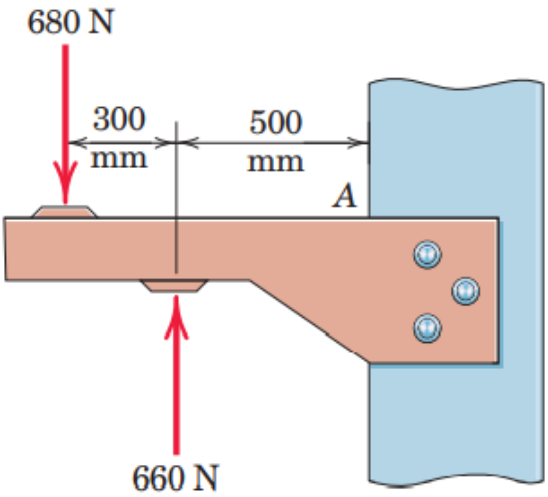
H.W.1:

Determine and locate the resultant **R** of the two forces and one couple acting on the I-beam.



H.W.2:

Where does the resultant of the two forces act?



H.W.3:

Determine the resultant **R** of the three tension forces acting on the eye bolt. Find the magnitude of **R** and the angle which **R** makes with the positive x-axis.

