**Dept. of Medical Instrumentation Engineering**

**First Stage:** **Academic Year: 2020-2021**

**Subject: Mechanics** **Instructor: Assist. Prof. Dr. Saad Alkhfaji**

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# LECTURE 2:

## Rectangular Components

### Introduction: Newton's Laws:

Sir Isaac Newton was the first to state correctly the basic laws governing the motion of a particle and to demonstrate their validity. These laws are:
**Law I.** A particle remains at rest or continues to move with uniform velocity (in a straight line with a constant speed) if there is no unbalanced force acting on it.

**Law II.** The acceleration of a particle is proportional to the vector sum of forces acting on it, and is in the direction of this vector sum.

**Law III.** The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear (they lie on the same line).

Newton's second law forms the basis for most of the analysis in dynamics. As applied to a particle of mass m, it may be stated as:

$$\mathbf{F} = m\mathbf{a}$$

where **F** is the vector sum of forces acting on the particle and **a** is the resulting acceleration. This equation is a vector equation because the direction of **F** must agree with the direction of **a**, and the magnitudes of **F** and m**a** must be equal.

## Units:

In mechanics we use four fundamental quantities called dimensions. These are length, mass, force, and time. The units used to measure these quantities cannot all be chosen independently because they must be consistent with Newton's second law, as shown in Equation above. Although there are a number of different systems of units, *only the two systems* most commonly used in science and technology will be used. The four fundamental dimensions and their units and symbols in the two systems are summarized in the following table:

| | | SI UNITS | | U.S. CUSTOMARY UNITS | |
|---|---|---|---|---|---|
| QUANTITY | DIMENSIONAL SYMBOL | UNIT | SYMBOL | UNIT | SYMBOL |
| Mass | M | Base units: kilogram | kg | slug | — |
| Length | L | Base units: meter | m | Base units: foot | ft |
| Time | T | Base units: second | s | Base units: second | sec |
| Force | F | newton | N | Base units: pound | lb |

## SI Units:

The International System of Units, abbreviated SI (from the French, Système International d'Unités), is accepted in the United States and throughout the world, and is a modern version of the metric system.

## U.S. Customary Units:

The U.S. customary, or British system of units, also called the foot-pound-second (FPS) system, has been the common system in business and industry in English-speaking countries. Although this system will in time be replaced by SI units, for many more years engineers must be able to work with both SI units and FPS units, and both systems are used freely in Engineering Mechanics.

The standard value for gravitational acceleration g is its value at sea level and at a $45^{o}$ latitude. In the two systems these values are:

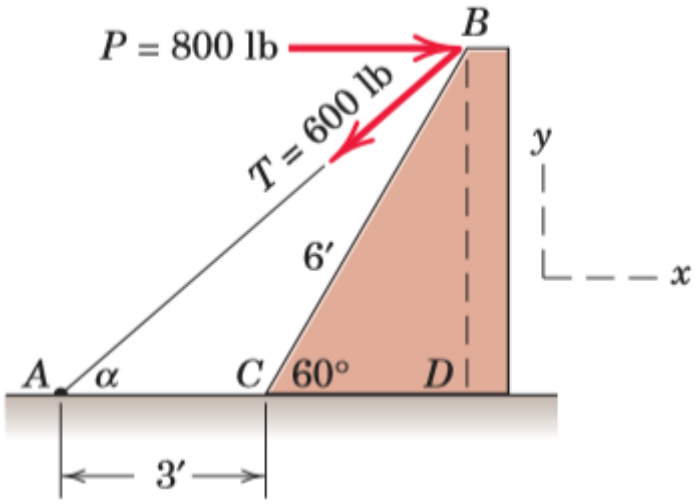
| | |
|---|---|
| SI units | $g = 9.806\ 65\ \text{m/s}^2$ |
| U.S. units | $g = 32.1740\ \text{ft/sec}^2$ |

The approximate values of 9.81 $\text{m/s}^2$ and 32.2 $\text{ft/sec}^2$, respectively, are sufficiently accurate for the vast majority of engineering calculations.

## Rectangular Components

### Example 1:

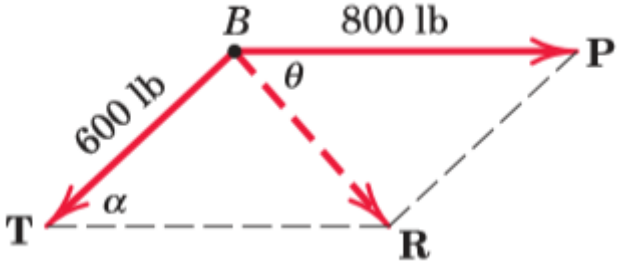
Combine the two forces **P** and **T**, which act on the fixed structure at B, into a single equivalent force **R**.




**Solution:**

**1- Graphical solution.**

The parallelogram for the vector addition of forces T and P is constructed as shown in Fig. below. The scale used here is 1 in. = 800 lb; a scale of 1 in. = 200 lb would be more suitable for regular-size paper and would give greater accuracy. Note that the angle α must be determined prior to construction of the parallelogram. From the given figure:



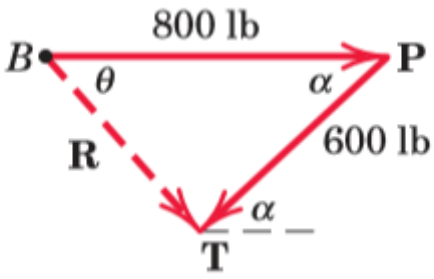
$$\tan \alpha = \frac{\overline{BD}}{\overline{AD}} = \frac{6 \sin 60°}{3 + 6 \cos 60°} = 0.866 \qquad \alpha = 40.9°$$

Measurement of the length $R$ and direction $\theta$ of the resultant force **R** yields the approximate results

$$R = 525 \text{ lb} \qquad \theta = 49° \qquad \textit{Ans.}$$

**2- Geometric solution.**

The triangle for the vector addition of **T** and **P** is shown in Fig. below. The angle α is calculated as above.




The law of cosines gives:

$$R^2 = (600)^2 + (800)^2 - 2(600)(800) \cos 40.9° = 274{,}300$$

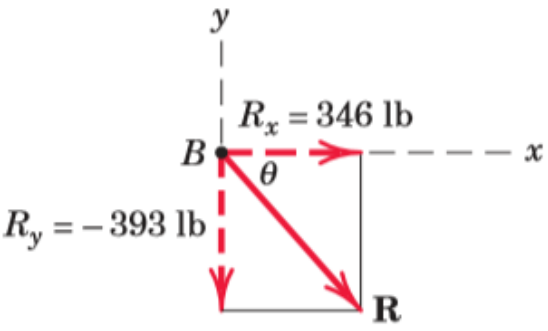
$$R = 524 \text{ lb} \qquad \textit{Ans.}$$

From the law of sines, we may determine the angle $\theta$ which orients **R**. Thus,

$$\frac{600}{\sin \theta} = \frac{524}{\sin 40.9°} \qquad \sin \theta = 0.750 \qquad \theta = 48.6° \qquad \textit{Ans.}$$

**3- Algebraic solution.**

By using the x-y coordinate system on the given figure, we may write:



$$R_x = \Sigma F_x = 800 - 600 \cos 40.9° = 346 \text{ lb}$$

$$R_y = \Sigma F_y = -600 \sin 40.9° = -393 \text{ lb}$$

The magnitude and direction of the resultant force **R** as shown in Fig. *c* are then

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(346)^2 + (-393)^2} = 524 \text{ lb} \qquad \textit{Ans.}$$

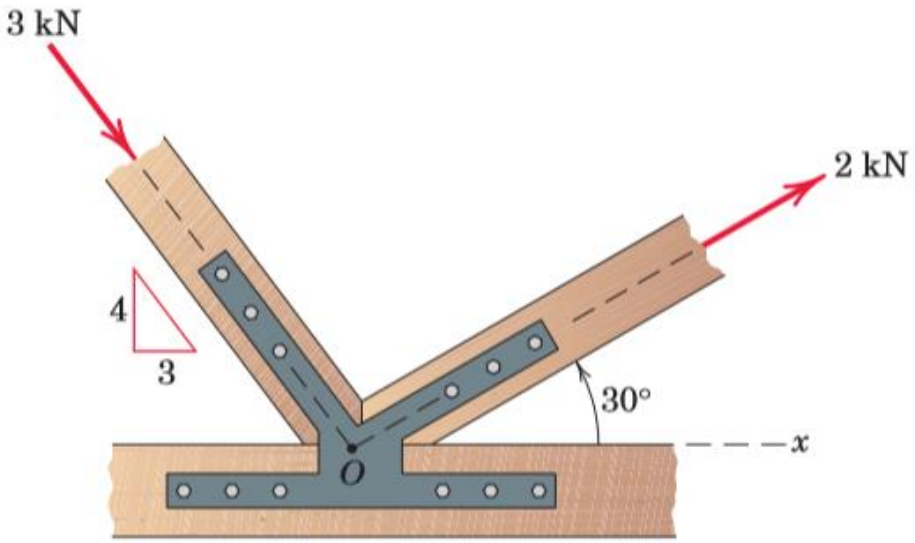
$$\theta = \tan^{-1} \frac{|R_y|}{|R_x|} = \tan^{-1} \frac{393}{346} = 48.6° \qquad \textit{Ans.}$$

The resultant **R** may also be written in vector notation as

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} = 346\mathbf{i} - 393\mathbf{j} \text{ lb} \qquad \textit{Ans.}$$

**H.W. 1:**

The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint *O*. Determine the magnitude of the resultant **R** of the two forces and the angle $\theta$ which **R** makes with the positive *x*-axis.

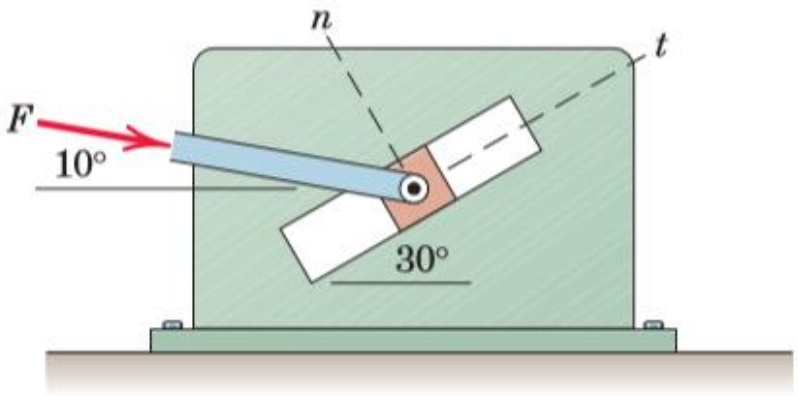



Ans.:

$$R = 3.80 \text{ kN}, \theta = 338° \text{ (or } -21.6°)$$

**H.W. 2:**

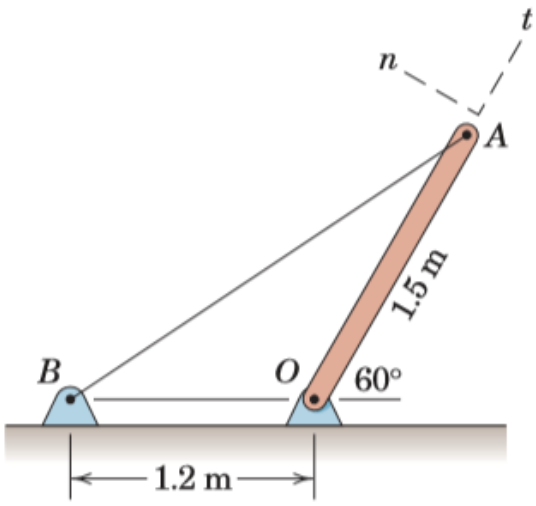
The $t$-component of the force $\mathbf{F}$ is known to be 75 N. Determine the $n$-component and the magnitude of $\mathbf{F}$.




Ans.: $F_n = -62.9 \text{ N}, F = 97.9 \text{ N}$

**H.W. 3:**

The cable $AB$ prevents bar $OA$ from rotating clockwise about the pivot $O$. If the cable tension is 750 N, determine the $n$- and $t$-components of this force acting on point $A$ of the bar.

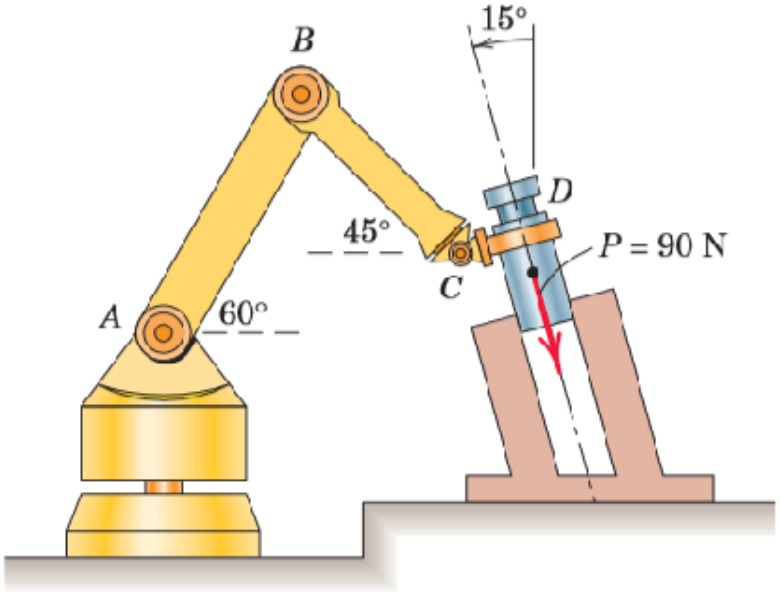



Ans.:

$$T_n = 333 \text{ N}, T_t = -672 \text{ N}$$

**H.W. 4:**

In the design of the robot to insert the small cylindrical part into a close-fitting circular hole, the robot arm must exert a 90-N force $P$ on the part parallel to the axis of the hole as shown. Determine the components of the force which the part exerts *on* the robot along axes ($a$) parallel and perpendicular to the arm $AB$, and ($b$) parallel and perpendicular to the arm $BC$.




Ans.:

$$BC: P_t = -77.9 \text{ N}, P_n = 45.0 \text{ N}$$
$$AB: P_t = 63.6 \text{ N}, P_n = 63.6 \text{ N}$$