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Transformer

Exact Equivalent circuit of loaded transformer

المحاضرة الثالثة

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Exact Equivalent Circuit of a Loaded Transformer

Fig. (1) shows the exact equivalent circuit of a transformer on load. Here R_1 is the primary winding resistance and R_2 is the secondary winding resistance. Similarly, X_1 is the leakage reactance of primary winding and X_2 is the leakage reactance of the secondary winding. The parallel circuit $R_0 - X_0$ is the no-load equivalent circuit of the transformer. The resistance R_0 represents the core losses (hysteresis and eddy current losses) so that current I_W which supplies the core losses is shown passing through R_0. The inductive reactance X_0 represents a loss-free coil which passes the magnetizing current I_m. The phasor sum of I_W and I_m is the no-load current I_0 of the transformer.

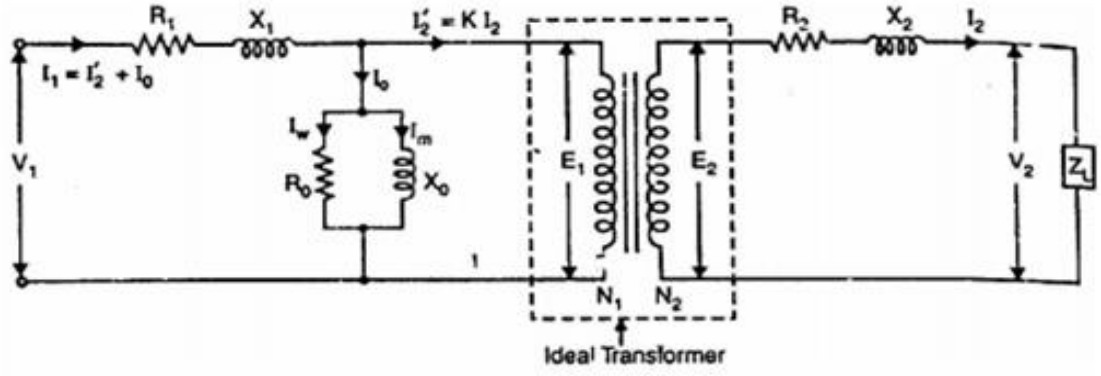


Fig. (1)

Note that in the equivalent circuit shown in Fig. (1), the imperfections of the transformer have been taken into account by various circuit elements. Therefore, the transformer is now the ideal one. Note that equivalent circuit has created two

normal electrical circuits separated only by an ideal transformer whose function is to change values according to the equation:

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I'_2}{I_2}$$

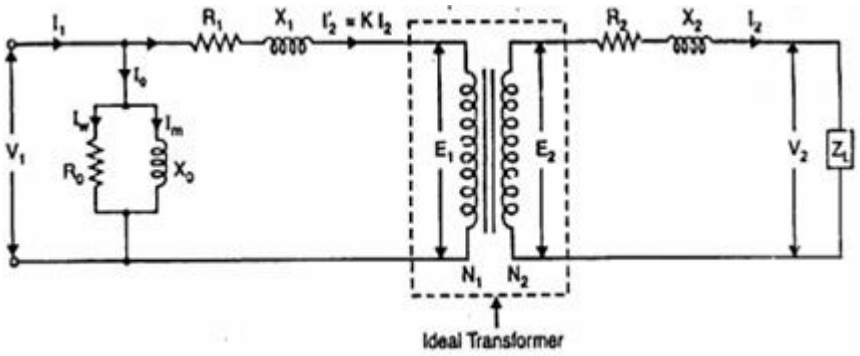
The following points may be noted from the equivalent circuit:

(i) When the transformer is on no-load (i.e., secondary terminals are open-circuited), there is no current in the secondary winding. However, the primary draws a small no-load current I_0. The no-load primary current I_0

Simplified Equivalent Circuit of a Loaded Transformer

The no-load current I_0 of a transformer is small as compared to the rated primary current. Therefore, voltage drops in R_1 and X_1 due to I_0 are negligible.

transferring the shunt circuit $R_0 - X_0$ to the input terminals as shown i



Equivalent circuit referred to primary

If all the secondary quantities are referred to the primary, we get the equivalent circuit of the transformer referred to the primary as shown in Fig. (5 (i)). This further reduces to Fig. (5 (ii)). Note that when secondary quantities are referred to primary, resistances/reactances/impedances are divided by K^2, voltages are divided by K and currents are multiplied by K.

$$\therefore \quad K_2 = \frac{R_2}{K^2}; \quad X'_2 = \frac{X_2}{K^2}; \quad Z'_L = \frac{Z_L}{K^2}; \quad V'_2 = \frac{V_2}{K}; \quad I'_2 = K I_2$$

$$Z_{01} = R_{01} + j X_{01}$$

where $\quad R_{01} = R_1 + R'_2; \quad X_{01} = X_1 + X'_2$

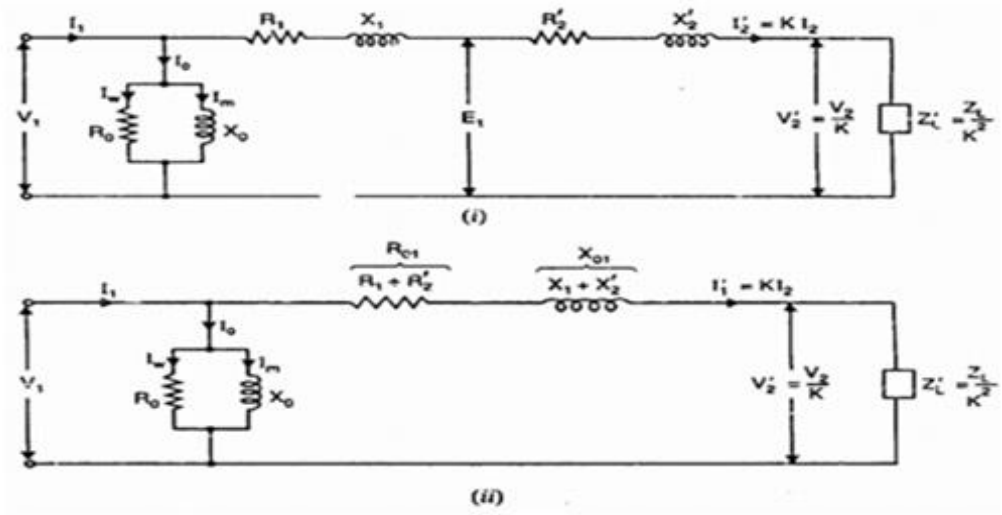


Fig. (5)

(a) magnetizing current (I_m) to create magnetic flux in the core and (b) the current I_W required to supply the core losses.

(ii) Equivalent circuit referred to secondary.

If all the primary quantities are referred to secondary, we get the equivalent circuit of the transformer referred to secondary as shown in Fig. (6 (i)). This further reduces to Fig. (6 (ii)). Note that when primary quantities are referred to secondary resistances/reactances/impedances are multiplied by K^2, voltages are multiplied by K. and currents are divided by K.

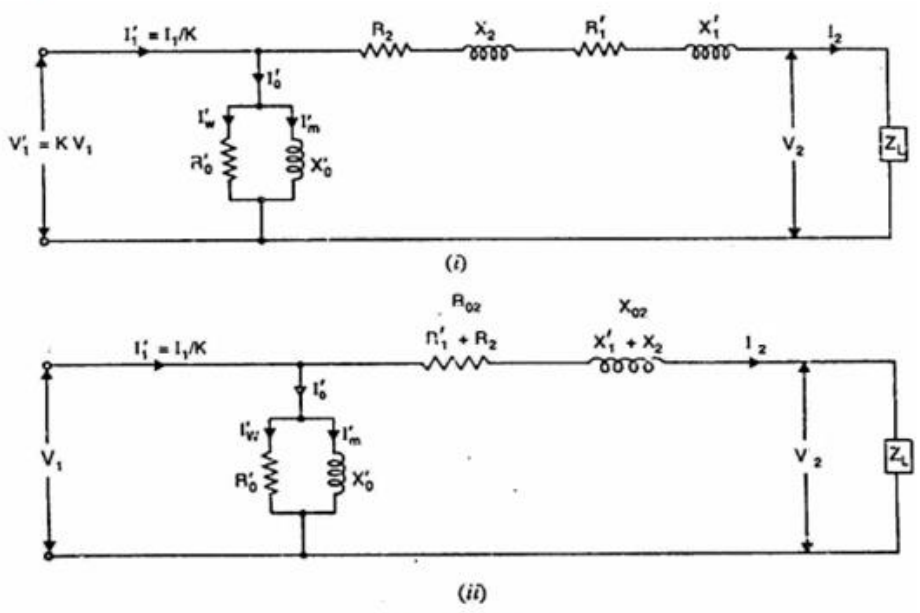


Fig. (6)

$$\therefore \quad R'_1 = K^2 R_1; \qquad X'_1 = K^2 X_1; \qquad V'_2 = K\,V_1; \quad I'_1 = \frac{I_1}{K}$$

$$Z_{02} = R_{02} + j\,X_{02}$$

where $\quad R_{02} = R_2 + R'_1; \quad X_{02} = X_2 + X'_1$

Approximate Equivalent Circuit of a Loaded Transformer

The no-load current I_0 in a transformer is only 1-3% of the rated primary current and may be neglected without any serious error. The transformer can then be shown as in Fig. (7). This is an approximate representation because no-load current has been neglected. Note that all the circuit elements have been shown external so that the transformer is an ideal one.

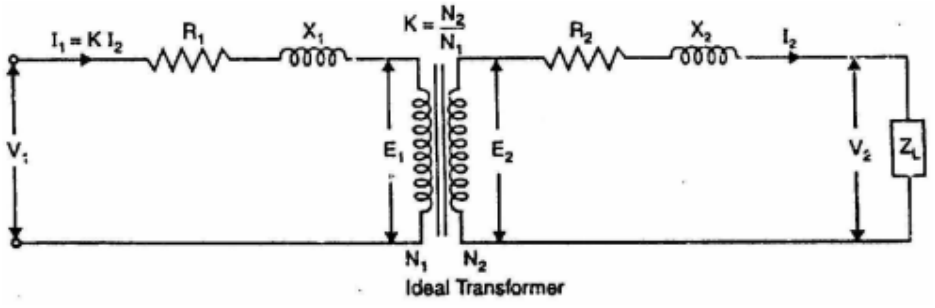


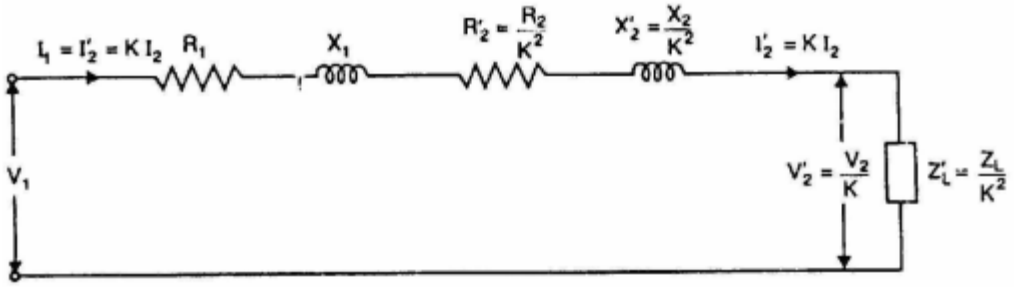
Fig. (7.)

(i) Equivalent circuit of transformer referred to primary

If all the secondary quantities are referred to the primary, we get the equivalent circuit of the transformer referred to primary as shown in Fig. (7.26). Note that when secondary quantities are referred to primary, resistances/reactances are divided by K^2, voltages are divided by K and currents are multiplied by K.

Actual secondary voltage, $V_2 = K\, V'_2$

Actual secondary current, $I_2 = I'_2/K$



(ii) Equivalent circuit of transformer referred to secondary

If all the primary quantities are referred to secondary, we get the equivalent circuit of the transformer referred to secondary as shown in Fig. (8). Note that when primary quantities are referred to secondary, resistances/reactances are multiplied by K^2, voltages are multiplied by K and currents are divided by K.

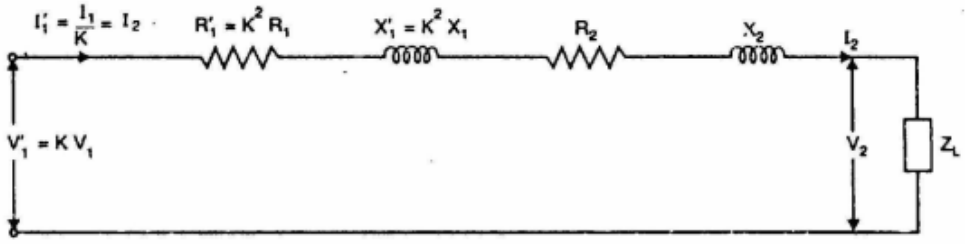


Fig. (8)

The equivalent circuit shown in Fig. (8) is an electrical circlet and can be solved for various voltages and currents. Thus is we find V'_1 and I'_1, then actual primary values can be determined as under:

Actual primary voltage, $V_1 = V'_1/K$

Actual primary current, $I_1 = KI'_1$

Example 1 . *A 4500/16000 V, 1500 kVA, 50 Hz transformer has the following parameters :*

$$R_1 = 0{\cdot}03\ \Omega,\quad R_2 = 0{\cdot}44\ \Omega,\quad R_0 = 1688\ \Omega$$
$$X_1 = 0{\cdot}092\ \Omega,\quad X_2 = 1{\cdot}34\ \Omega,\quad X_0 = 256\ \Omega$$

The transformer is supplying full-load at a power factor of 0·8 lagging. Using exact equivalent circuit, find the input current.

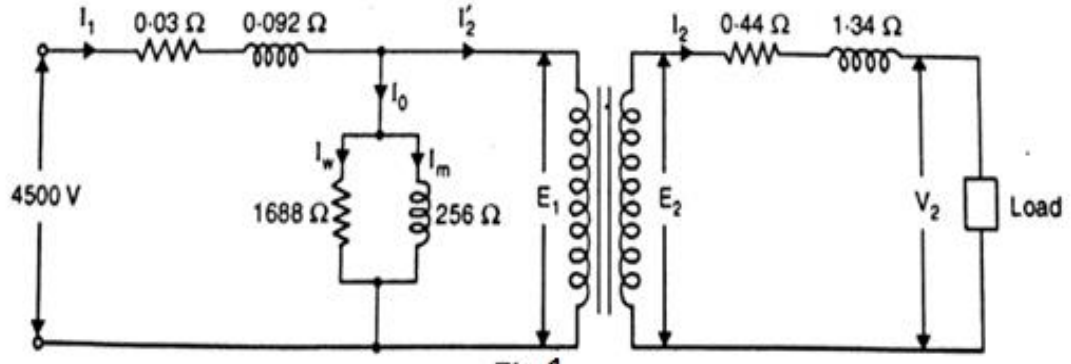


Fig. 1

Solution. Fig. 8.34 shows the exact equivalent circuit of the transformer. Here $K = 16000/4500 = 160/45$. Taking the load voltage as reference phasor, we have, $\mathbf{V}_2 = 16000\ \angle 0°$ V.

$$\text{F.L. secondary current, } I_2 = \frac{\text{kVA} \times 10^3}{V_2} = \frac{1500 \times 10^3}{16000} = 93{\cdot}75\ \text{A}$$

$$\therefore \quad \mathbf{I}_2 = 93{\cdot}75\ \angle -36{\cdot}9°\ \text{A} \qquad (\because \text{Load is at 0·8 p.f. lagging})$$

$$\mathbf{Z}_2 = (0{\cdot}44 + j\,1{\cdot}34)\ \Omega = 1{\cdot}41\ \angle 71{\cdot}82°\ \Omega$$

$$\mathbf{E}_2 = \mathbf{V}_2 + \mathbf{I}_2\mathbf{Z}_2 = 16000\,(1 + j0) + 93{\cdot}75\ \angle -36{\cdot}9° \times 1{\cdot}41\ \angle 71{\cdot}82°$$
$$= 16000\ \angle 0° + 132{\cdot}19\ \angle 34{\cdot}92° = (16108 + j\,75{\cdot}7)\ \text{V}$$

Now $$\mathbf{E}_1 = \frac{\mathbf{E}_2}{K} = \frac{1}{K} \times \mathbf{E}_2 = \frac{45}{160} \times (16108 + j\,75{\cdot}7) = (4530{\cdot}4 + j\,21{\cdot}3)\ \text{V}$$

and $$\mathbf{I}_2' = K\,\mathbf{I}_2 = \frac{160}{45} \times 93{\cdot}75\ \angle -36{\cdot}9° = 333{\cdot}3\ \angle -36{\cdot}9°\ \text{A} = (266{\cdot}64 - j\,200)\ \text{A}$$

$$\mathbf{I}_w = \frac{\mathbf{E}_1}{(1688 + j\,0)} = \frac{(4530{\cdot}4 + j\,21{\cdot}3)}{(1688 + j\,0)} = (2{\cdot}684 + j\,0{\cdot}013)\ \text{A}$$

$$\mathbf{I}_m = \frac{\mathbf{E}_1}{(0 + j\,256)} = \frac{(4530{\cdot}4 + j\,21{\cdot}3)}{(0 + j\,256)} = (0{\cdot}083 - j\,17{\cdot}7)\ \text{A}$$

$$\mathbf{I}_0 = \mathbf{I}_w + \mathbf{I}_m = (2{\cdot}684 + j\,0{\cdot}013) + (0{\cdot}083 - j\,17{\cdot}7) = (2{\cdot}77 - j\,17{\cdot}69)\ \text{A}$$

$$\therefore \quad \mathbf{I}_1 = \mathbf{I}_2' + \mathbf{I}_0 = (266{\cdot}64 - j\,200) + (2{\cdot}77 - j\,17{\cdot}69)$$
$$= (269{\cdot}4 - j\,217{\cdot}7)\ \text{A} = \mathbf{346{\cdot}4\ \angle -38{\cdot}94°\ A}$$

EXAMPLE 2 *A 10 kVA, single-phase transformer for 2000 / 400 V at no load, has* $R_1 = 5.5\,\Omega$, $X_1 = 12\,\Omega$, $R_2 = 0.2\,\Omega$, $X_2 = 0.45\,\Omega$. *Determine the approximate value of the secondary voltage at full load, 0.8 power factor (lagging), when the primary applied voltage is 2000 V.*

K= 400/2000= $\frac{1}{5}$

$$R_{o_2} = R_2 + R_1\left(\frac{N2}{N1}\right)^2 = 0.2 + 5.5\left(\frac{1}{5}\right)^2 = 0.42\,\Omega$$

$$X_{o_2} = X_2 + X_1\left(\frac{2}{1}\right)^2 = 0.45 + 12\left(\frac{1}{5}\right)^2 = 0.93\,\Omega$$

$$\text{kVA} = \frac{V_2 I_2}{1000}$$

$$I_2 = \frac{1000 \times \text{kVA}}{V_2} = \frac{1000 \times 10}{400} = 25\text{ A}$$

Since $\cos\phi_2 = 0.8$, $\sin\phi_2 = 0.6$

$$E_2 = E_1 \cdot \frac{N2}{N1} = V_1 \frac{N2}{N1} = 2000 \times \frac{1}{5} = 400\text{ V}$$

$$E_2 = V_2 + I_2 R_{o_2} \cos\phi_2 + I_2 X_{o_2} \sin\phi_2$$

$$400 = V_2 + 25 \times 0.42 \times 0.8 + 25 \times 0.93 \times 0.6$$

$$V_2 = 400 - 8.4 - 13.95 = 377.65\text{ V}$$

Power Loss in Transformer

The power loss in transformer can be divided into two types namely the copper and the iron loss. The iron losses can be further classified into two types namely the hysteresis losses and the eddy current losses

1.Copper Power Losses in Transformer

The total losses that take place in the winding resistance of a transformer are known as the 'Copper losses'. These losses in a transformer should be kept as low as possible to increases the efficiency of the transformer

To reduce the copper losses, it is essential to reduce the resistance of primary and secondary winding coils of the transformer i.e. size of the winding conductor is selected very carefully. These are also known as the variable losses as these are dependent on the square of load current To determine the copper losses, short circuit test on transformer is performed.

The total **copper losses in transformer** are:

$$= I_1^2R_1 + I_2^2R_2 = I_1^2R_{01} = I_2^2R_{02}$$

Where, I_1 , I_2 = primary and secondary currents respectively,
R_1, R_2 = primary and secondary resistances respectively,
R_{01}, R_{02} = equivalent resistances referred to primary and secondary respectively.

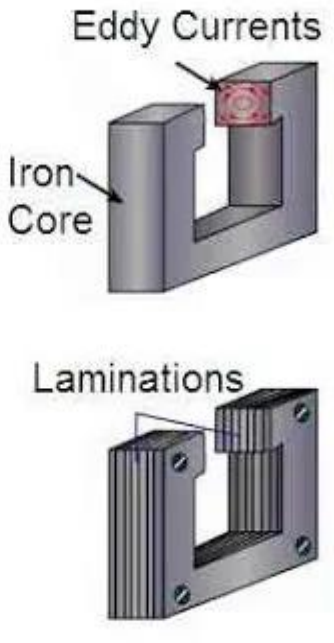
2.Iron Losses in Transformer

The power losses that take place in its iron core are nown as the ‘Iron losses’. These losses occur due to alternating flux set up in the core

These losses have two components named hysteresis losses and eddy current losses. To determine the iron losses, open circuit test of transformer is performed.

The hysteresis loss ($P_h = K_h V f B_m^{1.6}$) is frequency dependent. As we increase the frequency of operation, this loss increases proportionally.

Eddy Current Losses in Transformer



The core of transformer has some finite resistance. Hence due to the flow of eddy currents, some power losses take place and are known as 'Eddy current losses'

$(P_e = K_e V f t^2 B_m^2)$.

Example: The primary and the secondary windings of a 500kVA transformer have resistance of 0.42 ohms and 0.0011 ohms respectively.

The primary and the secondary voltages are 6600V and 400V respectively. Calculate copper losses at the full load and the half load.

Solution: Transformer rating = 500kVA

Primary resistance, $R_1 = 0.42\Omega$

Secondary resistance, $R_2 = 0.0011\Omega$

Primary voltage, $E_1 = 6600V$

Secondary voltage, $E_2 = 400V$

Transformation ratio, $K = E_2/E_1 = 400/6600 = 2/33$

Primary resistance referred to secondary,

$R_1' = K^2R_1 = (2/33)^2 \times 0.42 = 0.00154\ \Omega$

Total resistance referred to secondary, $R_{02} = R_2 + R_1' = 0.0011 + 0.00154$
$= 0.00264\ \Omega$

Full load secondary current, $I_2 = (kVA \times 10^3)/E_2 = (500 \times 10^3)/400 =$
1250A

Copper losses at full load $= I_2^2R_{02} = (1250)^2 \times 0.00264 = 4125W$

Secondary current at half load = 1250/2 = 625A

Copper losses at half load $= (625)^2 \times 0.00264 = 1031.25W$

Efficiency of transformer

Due to the losses in a transformer, the output power of a transformer is less than the nput power supplied.

$\therefore$ Power output = Power input – Total losses

$\therefore$ Power input = Power output + Total losses

= Power output + $P_i + P_{cu}$

The efficiency of any device is defined as the ratio of the power output to power nput. So for a transformer the efficiency can be expressed as,

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

$\therefore$

$$\eta = \frac{\text{Power output}}{\text{Power output} + P_i + P_{cu}}$$

Activate Wind
Go to Settings to a

Now Power output = $V_2 I_2 \cos\phi$

where $\cos\phi$ = Load power factor

The transformer supplies full load of current I_2 and with terminal voltage V_2.

P_{cu} = Copper losses on full load = $I_2^2 R_{o2}$

$$\therefore \quad \eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{o2}}$$

But $V_2 I_2$ = VA rating of a transformer

$$\therefore \quad \eta = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + I_2^2 R_{02}}$$

$$\% \eta = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + I_2^2 R_{02}} \times 100$$

Activa
Go to Se

This is full load percentage efficiency with,

I_2 = Full load secondary current

But if the transformer is subjected to fractional load then using the appropriate values of various quantities, the efficiency can be obtained.

Let n = Fraction by which load is less than full load = $\frac{\text{Actual load}}{\text{Full load}}$

In general for fractional load the efficiency is given by,

$$\% \eta = \frac{n(\text{VA rating}) \cos\phi}{n(\text{VA rating}) \cos\phi + P_i + n^2 (P_{cu})\text{F.L.}} \times 100$$

➡ **Example** : *A 4 kVA, 200/400 V, 50 Hz, single phase transformer has equivalent resistance referred to primary as 0.15 Ω Calculate,*

i) The total copper losses on full load.

ii) The efficiency while supplying full load at 0.9 p.f. lagging.

iii) The efficiency while supplying half load at 0.8 p.f. leading.

Assume total iron losses equal to 60 W.

Solution : The given values are,

$V_1 = 200$ V, $V_2 = 400$ V, $S = 4$ kVA , $R_{1e} = 0.15\ \Omega$, $P_i = 60$ W

$$K = \frac{400}{200} = 2$$

$$\therefore \quad R_{o2} = K^2 R_{o1} = (2)^2 \times 0.15 = 0.6\ \Omega$$

$$(I_2)\ \text{F.L.} = \frac{\text{kVA}}{V_2} = \frac{4\times10^3}{400} = 10\ \text{A}$$

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i) Total copper losses on full load,

$$(P_{cu})\ \text{F.L.} = [(I_2)\ \text{F.L.}]^2\ R_{o2} = (10)^2 \times 0.6 = \mathbf{60\ W}$$

ii) $\cos\phi = 0.9$ lagging and full load

$$\therefore \quad \%\,\eta = \frac{\text{VA rating} \cos\phi}{\text{VA rating} \cos\phi + P_i + (P_{cu})\,\text{F.L.}} \times 100$$

$$\therefore \quad \eta = \frac{4\times10^3 \times 0.9}{4\times10^3 \times 0.9 + 60 + 60} \times 100 = \mathbf{96.77\ \%}$$

iii) $\cos\phi = 0.8$ leading, half load

As half load, $n = 0.5$

$$(P_{cu})\text{H.L.} = n^2 \times (P_{cu})\text{F.L.} = (0.5)^2 \times 60 = 15\ \text{W}$$

$$\therefore \quad \%\,\eta = \frac{n \times (\text{VA rating}) \cos\phi}{n \times (\text{VA rating}) \cos\phi + P_i + (P_{cu})\,\text{H.L.}} \times 100$$

$$= \frac{0.5\times4\times10^3 \times 0.8}{0.5\times4\times10^3 \times 0.8 + 60 + 15} \times 100 = \mathbf{95.52\ \%}$$