Republic of Iraq Ministry of Higher
Education and Scientific Research
Ashur University
Department of Medical Instrumentation
Technical Engineering College

# Electrical technology

المرحلة الثانية

# Transformer

## Practical Transformer

## المحاضرة الثانية

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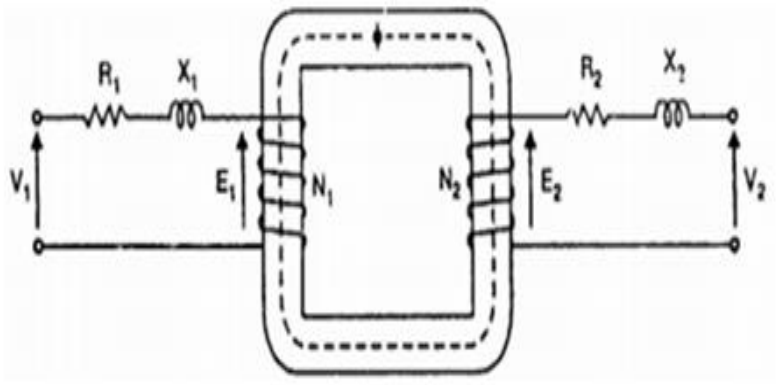
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## Practical Transformer

A practical transformer differs from the ideal transformer in many respects. The practical transformer has (i) iron losses (ii) winding resistances and (iii) magnetic leakage, giving rise to leakage reactance's

i) Iron losses. Since the iron core is subjected to alternating flux, there occurs eddy current and hysteresis loss in it. These two losses together are known as iron losses or core losses.

ii) Winding resistances. Since the windings consist of copper conductors, it immediately follows that both primary and secondary will have winding resistance. The primary resistance R1 and secondary resistance R2 act in series with the respective windings




(iii) **Leakage reactances**. Both primary and secondary currents produce flux. The flux $\phi$ which links both the windings is the useful flux and is called mutual flux. However, primary current would produce some flux $\phi$ which would not link the secondary winding as in fig Similarly, secondary current would produce some flux $\phi$ that would not link the primary winding. The flux such as $\phi_1$ or $\phi_2$ which links only one winding is called leakage flux. The leakage flux paths are mainly through the air.

## Practical Transformer on Load

### (i) No winding resistance and leakage flux

Fig. (7.8) shows a practical transformer with the assumption that resistances and leakage reactances of the windings are negligible. With this assumption, $V_2 = E_2$ and $V_1 = E_1$. Let us take the usual case of inductive load which causes the secondary current $I_2$ to lag the secondary voltage $V_2$ by $\phi_2$. The total primary current $I_1$ must meet two requirements viz.

(a) It must supply the no-load current $I_0$ to meet the iron losses in the transformer and to provide flux in the core.

(b) It must supply a current $I'_0$ to counteract the demagnetizing effect of secondary currently $I_2$. The magnitude of $I'_2$ will be such that:

$$N_1 I'_2 = N_2 I_2$$

or $$I'_2 = \frac{N_2}{N_1} I_2 = K I_2$$

The total primary current $I_1$ is the phasor sum of $I'_2$ and $I_0$ i.e.,

$$I_1 = I'_2 + I_0$$

where $$I'_2 = -K I_2$$

Note that $I'_2$ is 180° out of phase with $I_2$.

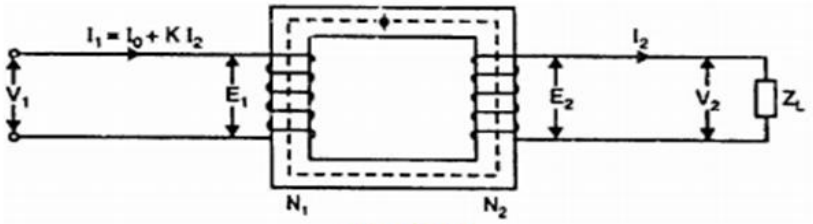



**Fig.(7.8)**

**Phasor diagram**. Fig. (7.9) shows the phasor diagram for the usual case of inductive load. Both $E_1$ and $E_2$ lag behind the mutual flux $\phi$ by 90°. The current $I'_2$ represents the primary current to neutralize the demagnetizing effect of secondary current $I_2$. Now $I'_2 = K I_2$ and is antiphase with $I_2$. $I_0$ is the no-load current of the transformer. The phasor sum of $I'_2$ and $I_0$ gives the total primary current $I_1$. Note that in drawing the phasor diagram, the value of K is assumed to be unity so that primary phasors are equal to secondary phasors.

Primary p.f. = $\cos \phi_1$

Secondary p.f. = $\cos \phi_2$
Primary input power = $V_1 I_1 \cos \phi_1$
Secondary output power = $V_2 I_2 \cos \phi_2$

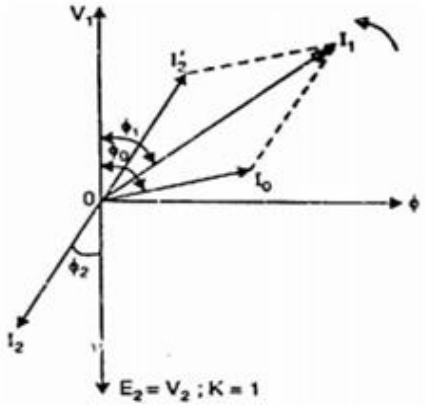



Fig.(7-9)

### (ii) Transformer with resistance and leakage reactance

Fig. (7.10) shows a practical transformer having winding resistances and leakage reactances. These are the actual conditions that exist in a transformer. There is voltage drop in $R_1$ and $X_1$ so that primary e.m.f. $E_1$ is less than the applied voltage $V_1$. Similarly, there is voltage drop in $R_2$ and $X_2$ so that secondary terminal voltage $V_2$ is less than the secondary e.m.f. $E_2$. Let us take the usual case of inductive load which causes the secondary current $I_2$ to lag behind the secondary voltage $V_2$ by $\phi_2$. The total primary current $I_1$ must meet two requirements viz.

(a) It must supply the no-load current $I_0$ to meet the iron losses in the transformer and to provide flux in the core.

(b) It must supply a current $I'_2$ to counteract the demagnetizing effect of secondary current $I_2$. The magnitude of $I'_2$ will be such that:

$$N_1 I'_2 = N_2 I_2$$

or
$$I'_2 = \frac{N_2}{N_1} I_2 = K I_2$$

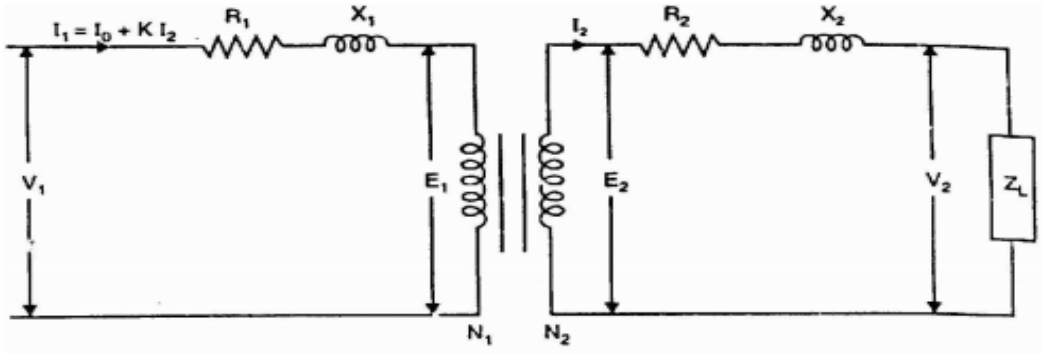



Fig.(7.10)

The total primary current $I_1$ will be the phasor sum of $I'_2$ and $I_0$ i.e.,

$$I_1 = I'_2 + I_0 \quad \text{where} \quad I'_2 = -KI_2$$

$$V_1 = -E_1 + I_1(R_1 + jX_1) \quad \text{where} \quad I_1 = I_0 + (-KI_2)$$

# Phasor diagram

Consider a transformer supplying the load as shown in the Fig. 1.

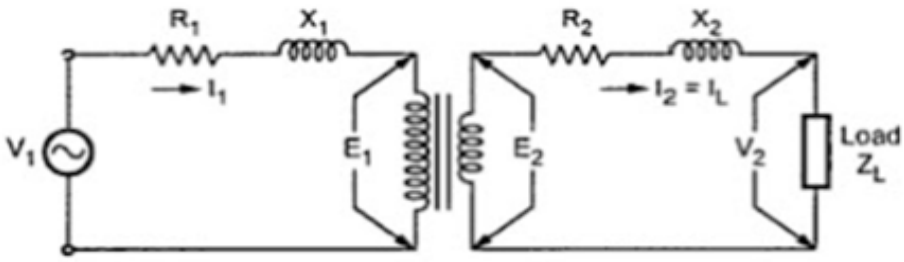



**Fig. 1**

The various transformer parameters are,

$R_1$ = Primary winding resistance

$X_1$ = Primary leakage reactance

$R_2$ = Secondary winding resistance

$X_2$ = Secondary leakage reactance

$Z_L$ = Load impedance

$I_1$ = Primary current

$V_1 = E_1 + I_1 Z_1$

$V_2 = E_2 - I_2(R_2 + JX_2)$

$V_2 = E_2 - I_2 Z_2$

$I_2'$ = Load component of current decided by the load

$I_2' = K I_2$ where K is transformer component

The primary voltage $V_1$ has now three components

The phasor diagram for the transformer on load

1. $-E_1$, the induced e.m.f. which opposes $V_1$
2. $I_1 R_1$, the drop across the resistance, in phase with $I_1$
3. $I_1 X_1$, the drop across the reactance, leading $I_1$ by 90°

$$\therefore \quad \bar{V}_1 = -\bar{E}_1 + \bar{I}_1\bar{R}_1 + \bar{I}_1\bar{X}_1 \quad \text{... phasor sum}$$

$$= -\bar{E}_1 + \bar{I}_1 (R_1 + j X_1)$$

$$\boxed{V_1 = -\bar{E}_1 + \bar{I}_1\bar{Z}_1}$$

The secondary induced e.m.f. has also three components,

1. $V_2$, the terminal voltage across the load
2. $I_2 R_2$, the drop across the resistance, in phase with $I_2$
3. $I_2 X_2$, the drop across the reactance, leading $I_2$ by 90°

$$\therefore \quad \bar{E}_2 = \bar{V}_2 + \bar{I}_2\bar{R}_2 + \bar{I}_2\bar{X}_2 \quad \text{... phasor sum}$$

$$\therefore \quad \bar{V}_2 = \bar{E}_2 - \bar{I}_2 (R_2 + j X_2)$$

$$\therefore \quad \boxed{V_2 = \bar{E}_2 - \bar{I}_2\bar{Z}_2}$$

**phasor diagram for the transformer on load**

## Unity power factor load, $\cos\Phi_2 = 1$

As load power factor is unity, the voltage $V_2$ and $I_2$ are in phase. Steps to draw the phasor diagram are,

1. Consider flux Φ as reference
2. $E_1$ lags Φ by 90°. Reverse $E_1$ to get $-E_1$.
3. $E_1$ and $E_2$ are inphase
4. Assume $V_2$ in a particular direction
5. $I_2$ is in phase with $V_2$.
6. Add $I_2 R_2$ and $I_2 X_2$ to to get $E_2$.
7. Reverse $I_2$ to get $I_2'$.
8. Add $I_o$ and $I_2'$ to get $I_1$.
9. Add $I_1 R_1$ and to $-E_1$ to get $V_1$.

Angle between $V_1$ and $I_1$ is $\Phi_1$ and $\cos\Phi_1$ is primary power factor. Remember that $I_1X_1$ leads $I_1$direction by 90° and $I_2 X_2$ leads

$I_2$ by 90° as current through inductance lags voltage across inductance by 90°. The phasor diagram is shown in the Fig.2

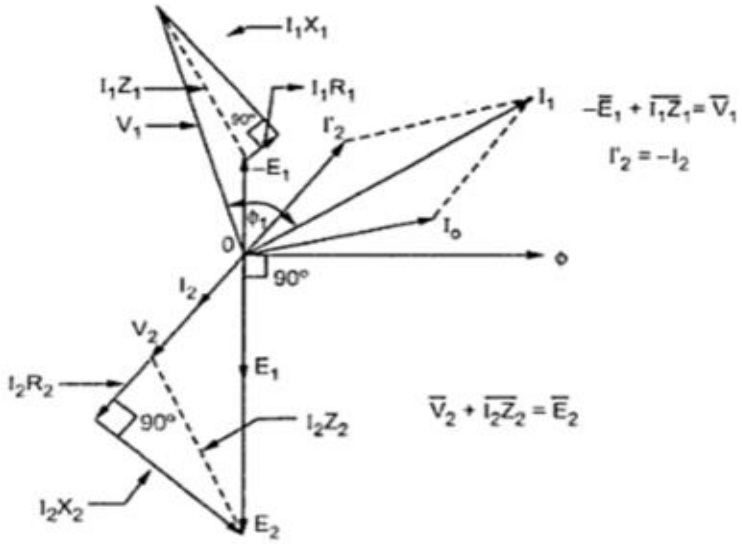



**Fig. 2 Phasor diagram for unity power factor load**

## Lagging Power Factor Load, cos $\Phi_2$:

As load power factor is lagging cos$\Phi_2$, the current $I_2$ lags $V_2$ by angle $\Phi_2$. So only changes in drawing the phasor diagram is to draw $I_2$ lagging $V_2$ by $\Phi_2$ in step 5 discussed earlier. Accordingly direction of $I_2R_2$, $I_2X_2$, $I_2'$, $I_1$, $I_1R_1$ and $I_1X_1$ will change. Remember that whatever may be the power factor of load, $I_2X_2$ leads $I_2$ by 90° and $I_1X_1$ leads $I_1$ by 90°. The complete phasor diagram is shown in the Fig. 3.

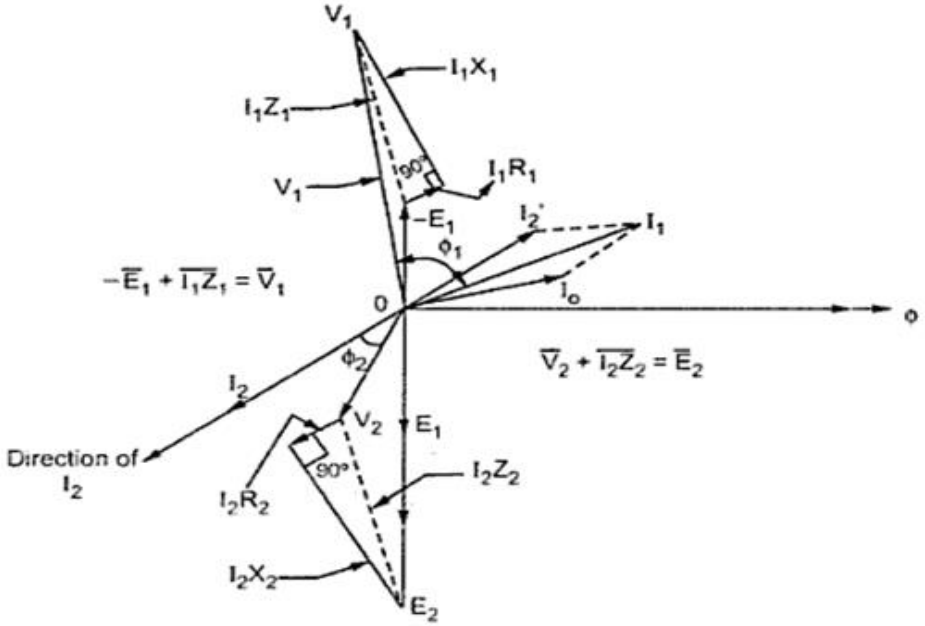



**Fig. 3 Phasor diagram for lagging power factor**

## Shifting Impedances in A Transformer

Fig. (2 ) shows a transformer where resistances and reactances are shown external to the windings. The resistance and reactance of one winding can be transferred to the other by appropriately using the factor $K^2$. This makes the analysis of the transformer a simple affair because then we have to work in one winding only.

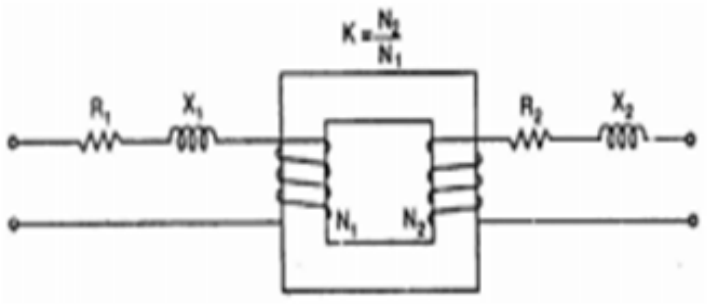



Fig.( 2 )

i.e., impedance ratio ($Z_2/Z_1$) is equal to the square of voltage transformation ratio. In other words, an impedance $Z_2$ in secondary becomes $Z_2/K^2$ when transferred to primary. Likewise, an impedance $Z_1$ in the primary becomes $K^2 Z_1$ when transferred to the secondary.

Similarly, $\frac{R_2}{R_1} = K^2$ and $\frac{X_2}{X_1} = K^2$

Note the importance of above relations. We can transfer the parameters from one winding to the other. Thus:

(i) A resistance $R_1$ in the primary becomes $K^2 R_1$ when transferred to the secondary.

(ii) A resistance $R_2$ in the secondary becomes $R_2/K^2$ when transferred to the primary.

(iii) A reactance $X_1$ in the primary becomes $K^2 X_1$ when transferred to the secondary.

(iv) A reactance $X_2$ in the secondary becomes $X_2/K^2$ when transferred to the primary.

***Note***: It is important to remember that:

(i) When transferring resistance or reactance from primary to secondary, multiply it by $K^2$.

(ii) When transferring resistance or reactance from secondary to primary, divide it by $K^2$.

(iii) When transferring voltage or current from one winding to the other, only K is used.

## (i) Referred to primary

When secondary resistance or reactance is transferred to the primary, it is divided by $K^2$. It is then called equivalent secondary resistance or reactance referred to primary and is denoted by $R'_2$ or $X'_2$.

Equivalent resistance of transformer referred to primary

$$R_{01} = R_1 + R'_2 = R_1 + \frac{R_2}{K^2}$$

Equivalent reactance of transformer referred to primary

$$X_{01} = X_1 + X'_2 = X_1 + \frac{X_2}{K^2}$$

Equivalent impedance of transformer referred to primary

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$

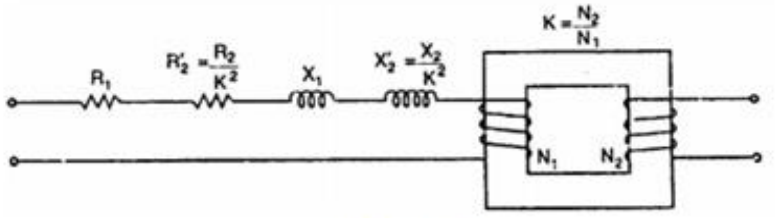



**Fig. ( 3 )**

Fig. ( 3 ) shows the resistance and reactance of the secondary referred to the primary. Note that secondary now has no resistance or reactance.

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## Impedance Ratio

Consider a transformer having impedance $Z_2$ in the secondary as shown in Fig. ( 1 ).

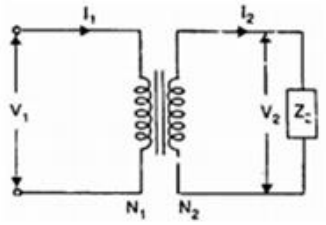



**Fig. ( 1 )**

$$Z_2 = \frac{V_2}{I_2}$$

$$Z_1 = \frac{V_1}{I_1}$$

$$\therefore \quad \frac{Z_2}{Z_1} = \left(\frac{V_2}{V_1}\right) \times \left(\frac{I_1}{I_2}\right)$$

or

$$\frac{Z_2}{Z_1} = K^2$$

## (ii) Referred to secondary

When primary resistance or reactance is transferred to the secondary, it is multiplied by $K^2$. It is then called equivalent primary resistance or reactance referred to the secondary and is denoted by $R'_1$ or $X'_1$.

Equivalent resistance of transformer referred to secondary

$$R_{02} = R_2 + R'_1 = R_2 + K^2 R_1$$

Equivalent reactance of transformer referred to secondary

$$X_{02} = X_2 + X'_1 = X_2 + K^2 X_1$$

Equivalent impedance of transformer referred to secondary

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$

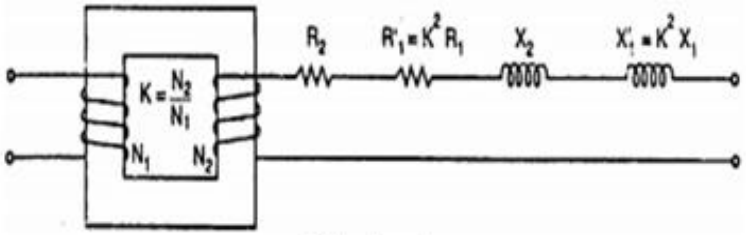



Fig. ( 4 )

Fig. ( 4 shows the resistance and reactance of the primary referred to the secondary. Note that primary now has no resistance or reactance.

Example 1.A 30 kVA, 2000/200V, single-phase, 50 Hz transformer has a primary resistance of 3.5 and reactance 4.5Ω.The secondary resistance and reactance are 0.015 Ω and 0.02 Ω respectively. Find:

)i) Equivalent resistance, reactance and impedance referred to primary.

)ii) Equivalent resistance, reactance and impedance referred to secondary.

)iii) Total copper loss of the transformer.

Solution.

Primary resistance, R1 = 3.5 Ω

Primary reactance, X1 = 4.5 Ω

Secondary resistance, R2 = 0.015 Ω

Secondary reactance, X2 = 0.02 Ω

**Transformation ratio,** $K = \frac{200}{2000} = \frac{1}{10}$

**(i) Equivalent resistance referred to primary,**

$$R_{01} = R_1 + R'_2 = R_1 + \frac{R_2}{K^2} = 3.5 + \frac{0.015}{\left(\frac{1}{10}\right)^2} = 5.0\ \Omega \quad (Ans.)$$

**Equivalent reactance referred to primary,**

$$X_{01} = X_1 + X'_2 = X_1 + \frac{X_2}{K^2} = 4.5 + \frac{0.02}{\left(\frac{1}{10}\right)^2} = 6.5\ \Omega \quad (Ans.)$$

**Equivalent impedance referred to primary,**

$$Z_{01} = \sqrt{(R_{01})^2 + (X_{01})^2} = \sqrt{(5)^2 + (6.5)^2} = 8.2\ \Omega. \quad (Ans.)$$

(ii) Equivalent resistance referred to secondary,

$$R_{o2} = R_2 + R_1' = R_2 + K^2 R_1$$

$$= 0.015 + \left(\frac{1}{10}\right)^2 \times 3.5 = \mathbf{0.05\,\Omega} \quad (\textit{Ans.})$$

Equivalent reactance referred to secondary,

$$X_{o2} = X_2 + X_1' = X_2 + K^2 X_1 = 0.02 + \left(\frac{1}{10}\right)^2 \times 4.5 = \mathbf{0.06.5\,\Omega} \quad (A$$

Equivalent impedance referred to secondary,

$$Z_{o2} = \sqrt{(R_{o2})^2 + (X_{o2})^2} = \sqrt{(0.05)^2 + (0.065)^2} = \mathbf{0.082\,\Omega.} \quad (\textit{Ans}$$

(iii) **Total copper loss:**

$$\text{Secondary current} = \frac{30 \times 1000}{200} = 150\ \text{A}$$

$$\text{copper loss} = I_2^2\, R_{02} = 150^2 \times 0.05 = \mathbf{1125\ W.}\ \text{(Total Ans.)}$$

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## Home work

Example: A 15KVA, 2200/110V transformer has $R_1$=1.75Ω, R2=0.0045 Ω the leakage reactance are $X_1$=2.6 Ω, $X_2$ =0.0075 Ω calculate,

(i) Equivalent resistance and reactance as referred to primary.

(ii) Equivalent resistance and reactance as referred to secondary.

(iii) Equivalent impedance referred to primary and secondary.

(iv) Total copper losses