2.1. Trigonometric Integrals

Some of the following trigonometry identities may be needed.

- A.) $\cos^2 x + \sin^2 x = 1$
- B.) $\sin 2x = 2 \sin x \cos x$
- C.) $\cos 2x = 2\cos^2 x - 1$ so that $\cos^2 x = \dfrac{1 + \cos 2x}{2}$
- D.) $\cos 2x = 1 - 2\sin^2 x$ so that $\sin^2 x = \dfrac{1 - \cos 2x}{2}$
- E.) $\cos 2x = \cos^2 x - \sin^2 x$
- F.) $1 + \tan^2 x = \sec^2 x$ so that $\tan^2 x = \sec^2 x - 1$
- G.) $1 + \cot^2 x = \csc^2 x$ so that $\cot^2 x = \csc^2 x - 1$

2.2 The direct integration for trig. Functions

- 1.) $\int \cos x \, dx = \sin x + C$
- 2.) $\int \sin x \, dx = -\cos x + C$
- 3.) $\int \sec^2 x \, dx = \tan x + C$
- 4.) $\int \csc^2 x \, dx = -\cot x + C$
- 5.) $\int \sec x \tan x \, dx = \sec x + C$
- 6.) $\int \csc x \cot x \, dx = -\csc x + C$
- 7.) $\int \tan x \, dx = \ln|\sec x| + C$
- 8.) $\int \cot x \, dx = \ln|\sin x| + C$
- 9.) $\int \sec x \, dx = \ln|\sec x + \tan x| + C$
- 10.) $\int \csc x \, dx = \ln|\csc x - \cot x| + C$

EXAMPLE Evaluate

$$\int \sin^3 x \cos^2 x \, dx.$$

Solution This is an example of Case 1.

$$\begin{aligned}
\int \sin^3 x \cos^2 x \, dx &= \int \sin^2 x \cos^2 x \sin x \, dx && m \text{ is odd.} \\
&= \int (1 - \cos^2 x) \cos^2 x \, (-d(\cos x)) && \sin x \, dx = -d(\cos x) \\
&= \int (1 - u^2)(u^2)(-du) && u = \cos x \\
&= \int (u^4 - u^2) \, du && \text{Multiply terms.} \\
&= \frac{u^5}{5} - \frac{u^3}{3} + C = \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C. && \blacksquare
\end{aligned}$$

EXAMPLE Evaluate

$$\int \cos^5 x \, dx.$$

Solution This is an example of Case 2, where $m = 0$ is even and $n = 5$ is odd.

$$\begin{aligned}
\int \cos^5 x \, dx &= \int \cos^4 x \cos x \, dx = \int (1 - \sin^2 x)^2 \, d(\sin x) && \cos x \, dx = d(\sin x) \\
&= \int (1 - u^2)^2 \, du && u = \sin x \\
&= \int (1 - 2u^2 + u^4) \, du && \text{Square } 1 - u^2. \\
&= u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + C = \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C. && \blacksquare
\end{aligned}$$

EXAMPLE Evaluate

$$\int_0^{\pi/4} \sqrt{1 + \cos 4x}\, dx.$$

Solution To eliminate the square root, we use the identity

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2} \quad \text{or} \quad 1 + \cos 2\theta = 2\cos^2 \theta.$$

With $\theta = 2x$, this becomes

$$1 + \cos 4x = 2\cos^2 2x.$$

Therefore,

$$\int_0^{\pi/4} \sqrt{1 + \cos 4x}\, dx = \int_0^{\pi/4} \sqrt{2\cos^2 2x}\, dx = \int_0^{\pi/4} \sqrt{2}\sqrt{\cos^2 2x}\, dx$$

$$= \sqrt{2}\int_0^{\pi/4} |\cos 2x|\, dx = \sqrt{2}\int_0^{\pi/4} \cos 2x\, dx \qquad \cos 2x \geq 0 \text{ on } [0, \pi/4]$$

$$= \sqrt{2}\left[\frac{\sin 2x}{2}\right]_0^{\pi/4} = \frac{\sqrt{2}}{2}[1 - 0] = \frac{\sqrt{2}}{2}. \quad \blacksquare$$

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