Solved examples

1) $\int \underset{u}{\ln x}\, \underset{dv}{dx}$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$\therefore \int \ln x\,dx = x\ln x - \int x\frac{1}{x}dx$

$= x\ln x - x + C$

2) $\int_1^{\pi} \ln x\,dx = x\ln x - x\Big]_1^{\pi} = \pi\ln\pi - \pi + 1$

3) $\int \underset{u}{x}\, \underset{dv}{e^{-x}dx}$

$u = x \qquad dv = e^{-x}dx$

$du = dx \qquad v = -e^{-x}$

$\therefore \int x\,e^{-x}dx = -xe^{-x} + \int e^{-x}dx$

$= -xe^{-x} - e^{-x} + C$

4) $\int_0^1 xe^{-x}dx = -xe^{-x} - e^{-x}\Big]_0^1$

$= -e^{-1} - e^{-1} - (0 - e^{0}) = 1 - \frac{2}{e}$

5) $\int \frac{(5x+2)}{x^2+2x-3}dx$

$\frac{5x+2}{x^2+2x-3} = \frac{A}{(x+3)} + \frac{B}{(x-1)}$

$5x+2 = A(x-1) + B(x+3)$

$5x + 2 = Ax - A + Bx + 3B$

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$A + B = 5 \longrightarrow A = 5 - B$

$-A + 3B = 2 \longrightarrow -(5-B) + 3B = 2$

$\therefore B = \frac{7}{2}$, $A = \frac{17}{2}$

$$\therefore \int \frac{5x+2}{x^2+2x-3} = \int \frac{17}{2(x+3)} + \int \frac{7}{2(x-1)}$$

$$= \frac{17}{2} \ln|x+3| + \frac{7}{2} \ln|x-1| + C$$

6) $\int \frac{(2x-1)\,dx}{(x+1)^2}$ =

$$\frac{2x-1}{(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2}$$

$2x-1 = A(x+1) + B$

$2x-1 = Ax + A + B$

$A = 2$, $A + B = -1 \longrightarrow B = -3$

$$\therefore \int \frac{(2x-1)\,dx}{(x+1)^2} = \int \frac{2\,dx}{(x+1)} + \int \frac{3\,dx}{(x+1)^2}$$

$$= 2\ln|x+1| + 3\frac{1}{(x+1)} + C$$

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7) $\int 2^x dx$

from $\int a^x dx = \frac{a^x}{\ln a} + C \quad a>0, a\neq 1$

then

$\int 2^x dx = \frac{2^x}{\ln 2} + C$

8) $\int_0^1 2^x dx = \frac{2^x}{\ln 2}\Big]_0^1 = \frac{2^1}{\ln 2} - \frac{2^0}{\ln 2}$

$= \frac{2}{\ln 2} - \frac{1}{\ln 2} = \frac{1}{\ln 2}$

9) $\int \frac{dx}{\sqrt{2-x^2}} =$ 1

from $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$ then

$\int \frac{dx}{\sqrt{2-x^2}} = \sin^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$

10) $\int \frac{dx}{9+x^2} = \int \frac{dx}{3^2+x^2}$

then from $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$ then

$= \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$

11) $\int \frac{dx}{\sqrt{x^2-8}} = \int \frac{dx}{\sqrt{x^2-(\sqrt{8})^2}} = \cosh^{-1}\left(\frac{x}{\sqrt{8}}\right) + C$

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