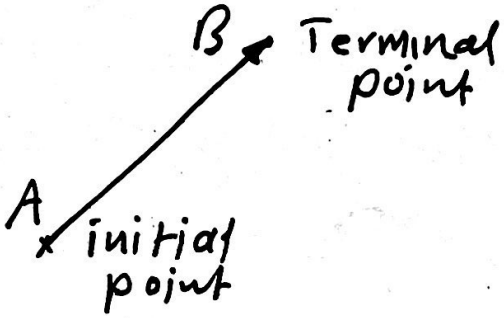
Vector:

Sometimes, a quantity is not enough to describe a measured value. It need direction in which it acts as well as how large it is, this is what called vector such as force, displacement and velocity

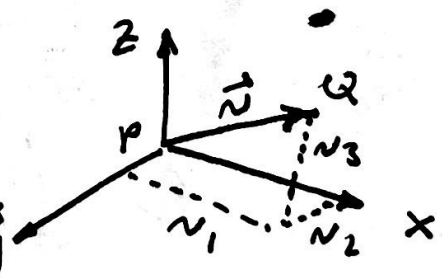


A vector in the plane is describe line segment $\overrightarrow{AB}$ which has initial point A and a terminal point B. Its length is denoted by $|\overrightarrow{AB}|$

If $\overrightarrow{AB} = \overrightarrow{CB}$ then the two vectors have the same length and direction.

$\overrightarrow{PQ}$ is a vector say has a name $\vec{N}$

Its components are N_1, N_2, N_3



then $\vec{N} = \langle N_1, N_2, N_3 \rangle$

where

$N_1 = x_2 - x_1$

$N_2 = y_2 - y_1$

$N_3 = z_2 - z_1$

where $Q(x_2, y_2, z_2)$ is a terminal point

$P(x_1, y_1, z_1)$ is initial point

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The magnitude or length of $\vec{v}$ is :

$$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Ex Find the (a) component (b) length of a vector with initial point $P(-3, 4, 1)$ and terminal point $Q(-5, 2, 2)$

Sol:

(a)

$$v_1 = x_2 - x_1 = -5 - (-3) = -2$$
$$v_2 = y_2 - y_1 = 2 - 4 = -2$$
$$v_3 = z_2 - z_1 = 2 - 1 = 1$$

then

$$\vec{v} = \langle -2, -2, 1 \rangle$$

(b)

$$|\vec{v}| = \sqrt{(-2)^2 + (-2)^2 + 1^2} = 3$$

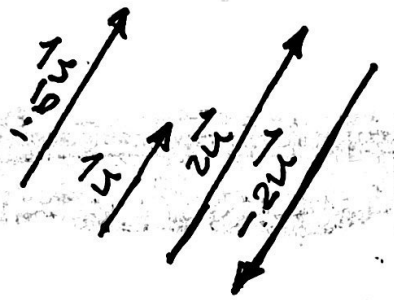
* Vector Addition and Multiplication by a scalar

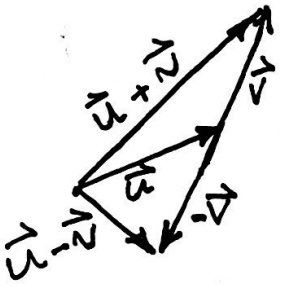
$$\vec{u} = \langle u_1, u_2, u_3 \rangle \; , \; \vec{v} = \langle v_1, v_2, v_3 \rangle$$

$$\vec{u} + \vec{v} = \langle u_1 + v_1 \, , \, u_2 + v_2 \, , \, u_3 + v_3 \rangle$$

$$c\vec{u} = \langle cu_1, cu_2, cu_3 \rangle$$

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$\vec{u}+\vec{v} = \langle u_1+v_1, u_2+v_2, u_3+v_3 \rangle$

$\vec{u}-\vec{v} = \langle u_1-v_1, u_2-v_2, u_3-v_3 \rangle$

Ex If $\vec{u}=\langle -1,3,1 \rangle$, $\vec{v}=\langle 4,7,0 \rangle$ Find

ⓐ $2\vec{u}+3\vec{v}$ ⓑ $\vec{u}-\vec{v}$ ⓒ $|\frac{1}{2}u|$

sol:

ⓐ $2\langle -1,3,1 \rangle + 3\langle 4,7,0 \rangle$

$= \langle -2,6,2 \rangle + \langle 12,21,0 \rangle$

$= \langle 10,27,2 \rangle$

ⓑ $\langle -1,3,1 \rangle - \langle 4,7,0 \rangle = \langle -5,-4,1 \rangle$

ⓒ $|\frac{1}{2}\vec{u}| = \langle -\frac{1}{2}, \frac{3}{2}, \frac{1}{2} \rangle = \sqrt{(-\frac{1}{2})^2+(\frac{3}{2})^2+(\frac{1}{2})^2} = \frac{1}{2}\sqrt{11}$

* properties of vector operation

① $\vec{u}+\vec{v} = \vec{v}+\vec{u}$

② $\vec{u}+0 = \vec{u}$

③ $0\vec{u} = \vec{0}$

④ $(\vec{u}+\vec{v})+\vec{w} = \vec{u}+(\vec{v}+\vec{w})$

⑤ $\vec{u}+(-\vec{u}) = 0$

⑥ $a(b\vec{u}) = (ab)\vec{u}$

⑦ $1\vec{u} = \vec{u}$

⑧ $a(\vec{u}+\vec{v}) = a\vec{u}+a\vec{v}$

⑨ $(a+b)\vec{u} = a\vec{u}+b\vec{v}$

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* Unit Vector

A vector of length one is called a unit vector

* any vector can be written as

$$\vec{N} = \langle N_1, N_2, N_3 \rangle$$

or

$$\vec{N} = N_1 i + N_2 j + N_3 K$$

where N_1 is the i component of the vector N_1
N_2 is the j component of the vector N_2
N_3 is the K component of the vector N_3

* If $P_1(x_1, y_1, z_1)$ the the initial point and $P_2(x_2, y_2, z_2)$ is the terminal point-

then

$$\vec{N} = \overrightarrow{P_1P_2} = (x_2 - x_1) i + (y_2 - y_1) j + (z_2 - z_1) K$$

and $\frac{\vec{N}}{|\vec{N}|}$ is a unit vector in the direction of $\vec{N}$ called the direction of the vector usually named $\vec{u}$.

Ex Find a vector with initial point $P_1(1,0,1)$ and terminal point $P_2(3,2,0)$.

sol:

$$\overrightarrow{P_1P_2} = (3-1) i + (2-0) j + (0-1) K$$

$$\overrightarrow{P_1P_2} = 2i + 2j - K$$

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Ex Find the component of the vector with initial point $P_1(1,0,1)$ and terminal point $P_2(3,2,0)$ also find its length.

sol:

$$\overrightarrow{P_1P_2} = \langle 3-1, 2-0, 0-1 \rangle$$

$$= \langle 2, 2, -1 \rangle$$

$$\text{length} = |\overrightarrow{P_1P_2}| = \sqrt{2^2+2^2+(-1)^2} = \sqrt{9} = 3$$

Ex Find a unit vector in the direction of the vector $\vec{v} = 2i + 2j - k$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{2i+2j-k}{\sqrt{2^2+2^2+(-1)^2}} = \frac{2}{3}i + \frac{2}{3}j - \frac{1}{3}k$$

* the equation $\vec{v} = |\vec{v}| \frac{\vec{v}}{|\vec{v}|}$ is vector expressed in terms of its length and direction.

Ex If $\vec{v} = 3i - 4j$ is a velocity vector, express $\vec{v}$ as a product of its length times a unit vector

$$|\vec{v}| = \sqrt{3^2+4^2} = 5$$

$$\text{unit vector } \frac{\vec{v}}{|\vec{v}|} = \frac{3i-4j}{5} = \frac{3}{5}i - \frac{4}{5}j$$

$$\vec{v} = 3i - 4j = 5\left(\frac{3}{5}i - \frac{4}{5}j\right)$$

HW solve Ex 12-2 pp 690 problems 1 to 16, 23 to 34

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