1.1.Set Notation

A **set** is a collection of objects. Each object in the set is called an **element** of the set. If an element x is a member of the set S, we write $x \in S$. If an element x is not a member of the set S, we write $x \notin S$. The **empty set** is the set that contains no elements. It is represented by {} or $\emptyset$. For sets A and B, A is called a **subset** of B, denoted $A \subseteq$ B, if every element of A is also an element of B.

The **union** of two sets A and B denoted $A \cup$ B is the set that consists of all elements that are **in *A* or in *B*** (or in both A and B). The **intersection** of two sets A and B, denoted $A \cap$ B is the set that consists of all elements that are in ***A* and in *B*.**

Sets are commonly defined in **roster notation** or **set-builder notation**. Roster notation provides a listing of each element in the set. For example, the set P of prime numbers between 0 and 10 (excluding 0 and 10) in roster notation is written as:

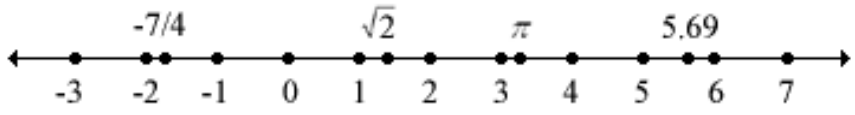
$P = \{ 2, 3, 5, 7 \}$

Note that prime numbers are the numbers dividable only by itself and 1. The **set-builder notation** defines a general element of the set and then provides a ruling for each element in the set. The previous example, in **set-builder notation** is written as:

$P = \{x \mid x \text{ is prime and } 0 < x < 10\}$

1.2 The Real Number Line

Every real number can be associated with a single point on the **real number line.**



1.3 Intervals

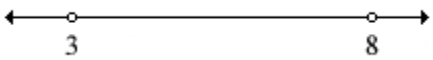
An **interval** is a set that consists of all real numbers between a given pair of numbers. It can also be thought of as a segment of the real number line.

An **endpoint** of an interval is either of the two points that mark the end of the line segment. An interval can include either endpoint, both endpoints or neither endpoint. To distinguish between these different intervals, we use interval notation.

An **open interval** does not include endpoints. The exclusion of the endpoints is indicated by round brackets () in interval notation. When the interval is represented by a segment of the real number line, the exclusion of an endpoint is illustrated by an open dot. For example, the interval of numbers between the integers 3 and 8, excluding 3 and 8, is written as

$(3, 8) = \{x \mid 3 < x < 8\}$

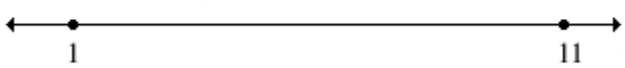
in interval notation. As a segment of the real number line, it would be represented by the line below.



A **closed interval** includes the endpoints. The inclusion of the endpoints is indicated by square brackets [] in interval notation. When the interval is represented by a segment of the real number line, the inclusion of an endpoint is illustrated by a closed dot. For example, the interval of numbers between the integers 1 and 11, including both 1 and 11, is written as

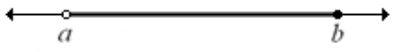
$[1, 11] = \{x \mid 1 \leq x \leq 11\}$

in interval notation. As a segment of the real number line, it would be represented by the line below.



One endpoint of an interval can be included, while the other is excluded. The interval $[a, b)$ represents all numbers between a and b, including a but not b. Similarly, the interval $(a, b]$ would represent all of the numbers between a and b, including b but not a. These intervals are shown in more detail in the table below.

Infinite intervals are those that do not have an endpoint in either the positive or negative direction or both. The interval extends forever in that direction. Infinite intervals are summarized in the table below.

Interval Notation	Number Line Sketch	Set-builder Notation
(a, b)		$\{x \mid a < x < b\}$
$(a, b]$		$\{x \mid a < x \le b\}$
$[a, b)$		$\{x \mid a \le x < b\}$
$[a, b]$		$\{x \mid a \le x \le b\}$
(a, ∞)		$\{x \mid x > a\}$
$(-\infty, b)$		$\{x \mid x < b\}$
$[a, \infty)$		$\{x \mid x \ge a\}$
$(-\infty, b]$		$\{x \mid x \le b\}$
$(-\infty, \infty)$		R

Let $A=\{x\in R|-2\le x\le 8\}$ *and* $B=\{x\in R|3<x<11\}$.
Find :

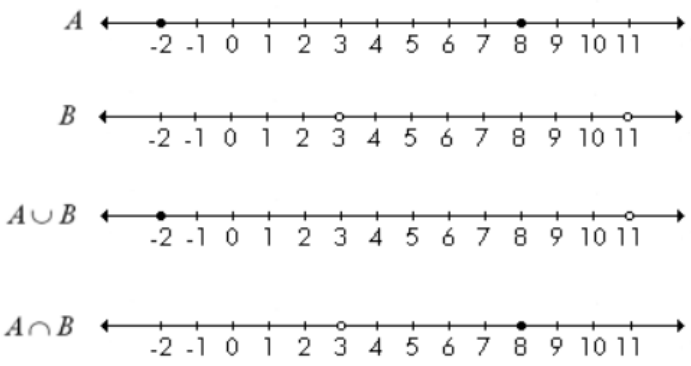
a) $A\cap B$
b) $A\cup B$

Solution:

$$a)\ A\cap B = \{x\in R|-2\le x\le 8 \text{ and } 3<x<11\}$$
$$= \{x\in R|3<x\le 8\}$$

$$b)\ A\cup B = \{x\in R|-2\le x\le 8 \text{ or } 3<x<11\}$$
$$= \{x\in R|-2\le x<11\}$$

The figures below show the number line representation of the intervals A, B, $A\cap B$ and $A\cup B$.



Solve as Homework

Ex let A={x∈ $R|-1\le x\le 7$} and B= x∈ $R|-3<x\le 9$}

Find A∩ B *and* $A\cup B$

1.3.2 Relations in Mathematics

In Maths, the relation is the relationship between two or more sets of values. Suppose, x and y are two sets of ordered pairs and set x has a relation with set y, then the values of set x are called domain whereas the values of set y are called range.

Example:

For ordered pairs={(1,2),(-3,4),(5,6),(-7,8),(9,2)}
The domain is = {-7,-3,1,5,9}
and range is = {2,4,6,8}

1.3.2. Types of Relations

There are eight types of relations:

1-Empty Relation

A relation R on a set A is said to be an empty relation if no element of set A is related to any other element of set A, , i.e. R = ∅.

Example

set A consists of only 100 hens in a poultry farm. Is there any possibility of finding a relation R of getting any elephant in the farm? No! R is a void or empty relation since there are only 100 hens and no elephants.

2-Universal Relation

A relation R is said to be a universal relation if every element of set A is related to every other element of set A, R = A × A

Example: If A is the set of students of grade 6 of a school then the relation R = { (a, b) | the difference in heights of a and b is less than 3 feet} is a universal relation.

3-Identity Relation

A relation R is said to be an identity relation if it contains only the ordered pairs where every element of set A is related to ONLY itself. i.e., R = { (a, a) }.

Example: If A = {1, 2, 3} then R = { (1, 1) (2, 2) (3, 3) } is the identity relationship.

4-Inverse Relation

The inverse relation of a relation R is denoted by R^{-1} and is obtained by interchanging the elements of each ordered pair of R.

Example: If R = { (1, 2) (3, 5) (5, 7) } then R^{-1} = { (2, 1) (5, 3) (7, 5) }.

5-Reflexive Relation

A relation R is said to be reflexive if all the elements of any set are mapped to itself.

Example: N is the set of all natural numbers and the relation R = { (a, b) | a = b} is a reflexive relation.

6-symmetric Relation

A relation R on a set A such that a_1, a_2 are elements of A, and if (a_1, a_2) ∈ R, and also if (a_2, a_1) ∈ R, then the relation R is a symmetric relation. A real-life example of this is, if John is a brother of Sam, then Sam can also be said to be a brother of John. The following is a math-related example of a symmetric relation.

Example: N is the set of all natural numbers and the relation R = { (a, b) | a = b} is a symmetric relation because whenever a = b, then it obviously means that b = a.

7-Transitive Relation

The relation R on set P, if (x,y)∈R and (y,z)∈R, then (x,z)∈ R for all a,b,c ∈ R is called transitive relation.

Example: N is the set of all natural numbers and the relation R = { (a, b) | a = b} is a transitive relation because whenever a = b and b = c then it obviously means that a = c.

8-Equivalence Relation

A relation R on a set A, if it is a reflexive, symmetric, and transitive relation, then it is called an equivalence relation.

Example: We have already seen that the relation R = { (a, b) | a = b} on the set of natural numbers is reflexive, symmetric, and transitive and hence it is an equivalence relation.

Q1. Let A be the set of two male persons in a family. R be a relation defined onset A is " is a brother of " , check whether R is symmetric or not.

Ans;

Let a,b are two persons in a family , then a,b ∈ A.

The given relation on set "is a brother of".

If a is the brother of b , then b is also the brother of a.

R={(a,b),(b,a)}

Hence, the relation R is symmetric.

Q2. Find the inverse relation of R={(1,2),(3,4),(5,6)}.

Ans.

Given relation R={(1,2),(3,4),(5,6)}

Consider R={(a,b):a ∈ P , b ∈ Q} be the relation from set P to set Q, then the relation from set Q to set P is known as inverse relation such that R^{-1} :Q→P={(a,b) ∈ R}.

So, inverse relation is obtained by taking the reverse of the given ordered pairs.

R^{-1}:{(2,1),(4,3),(6,5)}

Q3 Find the identity relation on the set P={x,y,z}

Ans the relation (I) is identity,then all the elements of set P are relate with itsself , such that for every a ∈ P, then (a,a) ∈ I

Given set is P={x,y,z}

Then the identity relation is given by I={(x,x),(y,y),(z,z)}

Q.4. Let $P = \{1, 2, 3\}$, R be a relation defined on set P as "is greater than" and $R = \{(2, 1), (3, 2), (3, 1)\}$. Verify R is transitive.

Ans:

Given set $P = \{1, 2, 3\}$,
Let $P = \{1, 2, 3\}$, then we have
$(b, a) = (2, 1) \rightarrow 2$ is greater than 1.
$(c, b) = (3, 2) \rightarrow 3$ is greater than 2.
$(c, a) = (3, 1) \rightarrow 3$ is greater than 1.
Thus in a transitive relation, if $(x, y) \in R$, $(y, z) \in R$, then $(x, z) \in R$.
So, the relation $R = \{(2, 1), (3, 2), (3, 1)\}$ is transitive.

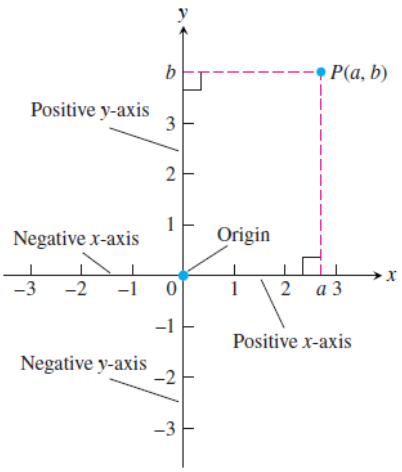
Q.5. Let set A contain the bag of red balls. Then the relation R of getting white balls from the bag of red balls is what type of relation?

Ans:

Given set A contains the bag contains the red balls. From the bag of red balls, relation R of getting the white ball is not possible. So, the given relation is an empty relation.

1.4. Cartesian Coordinates in the Plane

Points in the plane can be identified with ordered pairs of real numbers. On the horizontal *x*-axis, numbers are denoted by *x* and increase to the right. On the vertical *y*-axis, numbers are denoted by *y* and increase upward.



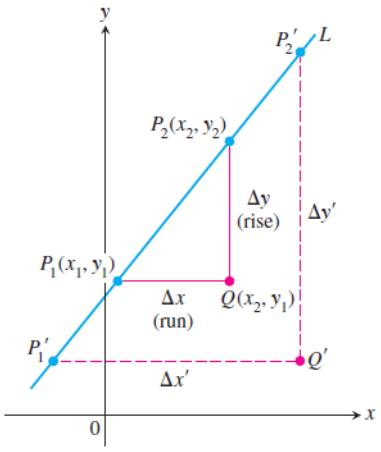
1.5 Equation of a line

DEFINITION **Slope**

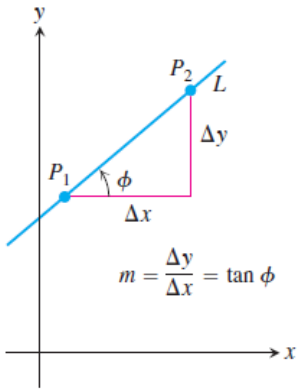
The constant

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

is the **slope** of the nonvertical line P_1P_2.



The relationship between the slope m of a non-vertical line and the line's angle of inclination ϕ is shown in Figure below



EXAMPLE Write an equation for the line through the point (2, 3) with slope $-3/2$.

Solution We substitute $x_1 = 2, y_1 = 3$, and $m = -3/2$ into the point-slope equation and obtain

$$y = 3 - \frac{3}{2}(x - 2), \quad \text{or} \quad y = -\frac{3}{2}x + 6.$$

When $x = 0, y = 6$ so the line intersects the y-axis at $y = 6$. ■

EXAMPLE A Line Through Two Points

Write an equation for the line through $(-2, -1)$ and $(3, 4)$.

Solution The line's slope is

$$m = \frac{-1 - 4}{-2 - 3} = \frac{-5}{-5} = 1.$$

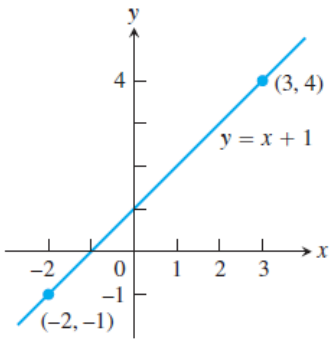
We can use this slope with either of the two given points in the point-slope equation:

With $(x_1, y_1) = (-2, -1)$
$y = -1 + 1 \cdot (x - (-2))$
$y = -1 + x + 2$
$y = x + 1$

With $(x_1, y_1) = (3, 4)$
$y = 4 + 1 \cdot (x - 3)$
$y = 4 + x - 3$
$y = x + 1$

Same result

Either way, $y = x + 1$ is an equation for the line (Figure). ■



In a special case, a slope is defined and the intersection of the line with the y-axis the equation of a line can be written as

$$y = mx + b$$

Where b is the distance from the origin (positive for upward and negative for downward)

EXAMPLE Finding the Slope and y-Intercept

Find the slope and y-intercept of the line $8x + 5y = 20$.

Solution Solve the equation for y to put it in slope-intercept form:

$$8x + 5y = 20$$
$$5y = -8x + 20$$
$$y = -\frac{8}{5}x + 4.$$

The slope is $m = -8/5$. The y-intercept is $b = 4$.

1.5.1 Parallel and perpendicular line

Lines that are parallel have equal angles of inclination, so they have the same slope (if they are not vertical). Conversely, lines with equal slopes have equal angles of inclination and so are parallel.

If two nonvertical lines L_1 and L_2 are perpendicular, their slopes m_1 and m_2 satisfy $m_1 m_2 = -1$, so each slope is the *negative reciprocal* of the other

1.6 Function

Functions are a tool for describing the real world in mathematical terms. A function can be represented by many methods such as an equation, a graph, or a numerical table.

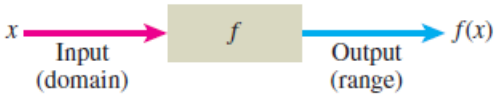
1.6.1. Domain and Range

The temperature at which water boils depends on the elevation above sea level (the boiling point drops as you ascend. The distance an object travels at constant speed along a straight-line path depends on the elapsed time. In each case, the value of one variable quantity, say y, depends on the value of another variable quantity, which we might call x. We say that "y is a function of x" and write this symbolically as

$$y = f(x) \qquad \text{"y equal f of x"}$$

the symbol f represents the function, the letter ***x* is the independent variable** representing the input value of f, and ***y* is the dependent variable** or output value of f at x.

The set D of all possible input values is called the domain of the function. The set of all values of f(x) as x varies throughout D is called the range R of the function



Example

show the domain and range of the following function

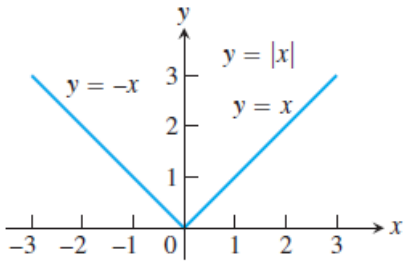
Function	Domain (x)	Range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$

1.6.2 Piecewise defined function

Sometimes a function is described by using different formulas on different parts of its domain. One example is the **absolute value function**

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0, \end{cases}$$

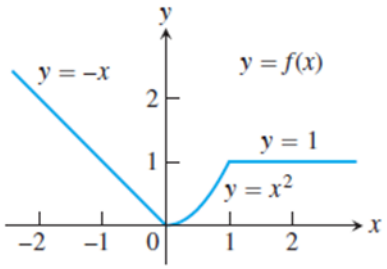
whose graph is given in Figure 1.8. The right-hand side of the equation means that the function equals x if $x \geq 0$, and equals $-x$ if $x < 0$. Here are some other examples.



EXAMPLE The function

$$f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

is defined on the entire real line but has values given by different formulas depending on the position of x. The values of f are given by $y = -x$ when $x < 0$, $y = x^2$ when $0 \leq x \leq 1$, and $y = 1$ when $x > 1$. The function, however, is *just one function* whose domain is the entire set of real numbers (Figure). ■



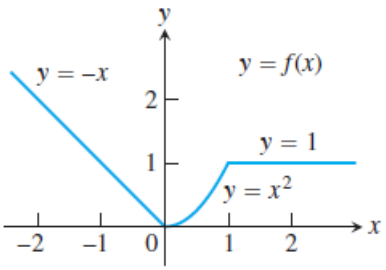
1.6.2 Increasing and decreasing functions

If the graph of a function *climbs* or *rises* as you move from left to right, we say that the function is *increasing*. If the graph *descends* or *falls* as you move from left to right, the function is *decreasing*.

DEFINITIONS Let f be a function defined on an interval I and let x_1 and x_2 be any two points in I.

1. If $f(x_2) > f(x_1)$ whenever $x_1 < x_2$, then f is said to be **increasing** on I.
2. If $f(x_2) < f(x_1)$ whenever $x_1 < x_2$, then f is said to be **decreasing** on I.

EXAMPLE The function graphed in Figure 1.9 is decreasing on $(-\infty, 0]$ and increasing on $[0, 1]$. The function is neither increasing nor decreasing on the interval $[1, \infty)$



1.6.3 Even Functions and Odd Functions: Symmetry

The graphs of *even* and *odd* functions have characteristic symmetry properties.

DEFINITIONS A function $y = f(x)$ is an

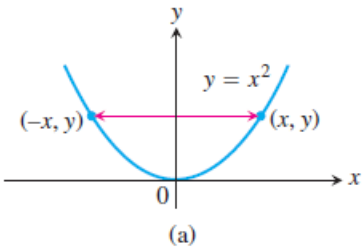
even function of x if $f(-x) = f(x)$,
odd function of x if $f(-x) = -f(x)$,

for every x in the function's domain.

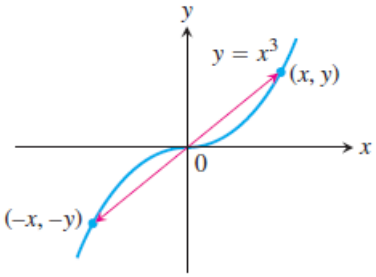
The names *even* and *odd* come from powers of x. If y is an even power of x, as in $y = x^2$ or $y = x^4$, it is an even function of x because $(-x)^2 = x^2$ and $(-x)^4 = x^4$. If y is an odd power of x, as in $y = x$ or $y = x^3$, it is an odd function of x because $(-x)^1 = -x$ and $(-x)^3 = -x^3$.

The graph of an even function is **symmetric about the y-axis**. Since $f(-x) = f(x)$, a point (x, y) lies on the graph if and only if the point $(-x, y)$ lies on the graph (Figure .a). A reflection across the y-axis leaves the graph unchanged.

The graph of an odd function is **symmetric about the origin**. Since $f(-x) = -f(x)$, a point (x, y) lies on the graph if and only if the point $(-x, -y)$ lies on the graph (Figure b). Equivalently, a graph is symmetric about the origin if a rotation of 180° about the origin leaves the graph unchanged. Notice that the definitions imply that both x and $-x$ must be in the domain of f.



(a)



(b)

EXAMPLE

$f(x) = x^2$	Even function: $(-x)^2 = x^2$ for all x; symmetry about y-axis.
$f(x) = x^2 + 1$	Even function: $(-x)^2 + 1 = x^2 + 1$ for all x; symmetry about y-axis (Figure 1.13a).
$f(x) = x$	Odd function: $(-x) = -x$ for all x; symmetry about the origin.
$f(x) = x + 1$	Not odd: $f(-x) = -x + 1$, but $-f(x) = -x - 1$. The two are not equal. Not even: $(-x) + 1 \neq x + 1$ for all $x \neq 0$ (Figure .). ■

1.7Composite Functions

DEFINITION If f and g are functions, the **composite** function $f \circ g$ ("f composed with g") is defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ consists of the numbers x in the domain of g for which $g(x)$ lies in the domain of f.

EXAMPLE If $f(x) = \sqrt{x}$ and $g(x) = x + 1$, find

(a) $(f \circ g)(x)$ **(b)** $(g \circ f)(x)$ **(c)** $(f \circ f)(x)$ **(d)** $(g \circ g)(x)$.

Solution

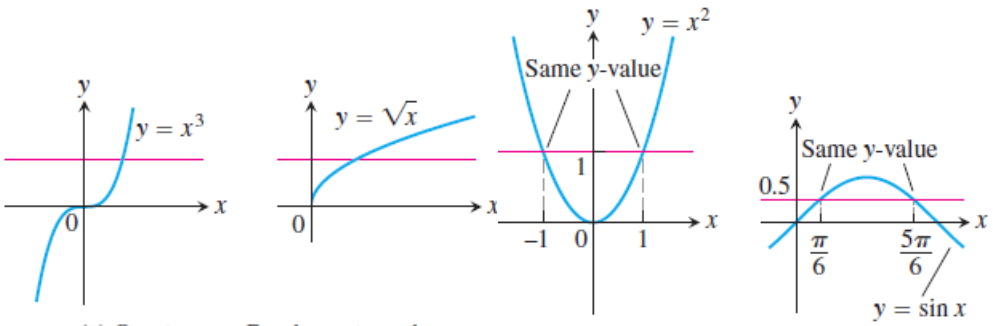
Composite	Domain
(a) $(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} = \sqrt{x+1}$	$[-1, \infty)$
(b) $(g \circ f)(x) = g(f(x)) = f(x) + 1 = \sqrt{x} + 1$	$[0, \infty)$
(c) $(f \circ f)(x) = f(f(x)) = \sqrt{f(x)} = \sqrt{\sqrt{x}} = x^{1/4}$	$[0, \infty)$
(d) $(g \circ g)(x) = g(g(x)) = g(x) + 1 = (x+1) + 1 = x + 2$	$(-\infty, \infty)$

To see why the domain of $f \circ g$ is $[-1, \infty)$, notice that $g(x) = x + 1$ is defined for all real x but belongs to the domain of f only if $x + 1 \geq 0$, that is to say, when $x \geq -1$. ■

1.8 Inverse Functions and Logarithms

A function that inverts the effect of a function f is called the *inverse* of f.

The Horizontal Line Test for One-to-One Functions
A function $y = f(x)$ is one-to-one if and only if its graph intersects each horizontal line at most once.



(a) One-to-one: Graph meets each horizontal line at most once.

(b) Not one-to-one: Graph meets one or more horizontal lines more than once.

Only a one-to-one function can have an inverse.To find the inverse of a function

1. Solve the equation $y = f(x)$ for x. This gives a formula $x = f^{-1}(y)$ where x is expressed as a function of y.
2. Interchange x and y, obtaining a formula $y = f^{-1}(x)$ where f^{-1} is expressed in the conventional format with x as the independent variable and y as the dependent variable.

EXAMPLE Find the inverse of $y = \frac{1}{2}x + 1$, expressed as a function of x.

Solution

1. *Solve for x in terms of y*: $y = \frac{1}{2}x + 1$

$$2y = x + 2$$

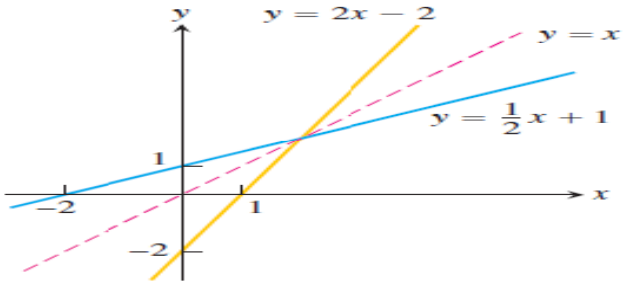
$$x = 2y - 2.$$

2. *Interchange x and y*: $y = 2x - 2$.

The inverse of the function $f(x) = (1/2)x + 1$ is the function $f^{-1}(x) = 2x - 2$. (See Figure . .) To check, we verify that both composites give the identity function:

$$f^{-1}(f(x)) = 2\left(\frac{1}{2}x + 1\right) - 2 = x + 2 - 2 = x$$

$$f(f^{-1}(x)) = \frac{1}{2}(2x - 2) + 1 = x - 1 + 1 = x.$$ ■



EXAMPLE Find the inverse of the function $y = x^2, x \geq 0$, expressed as a function of x.

Solution We first solve for x in terms of y:

$$y = x^2$$

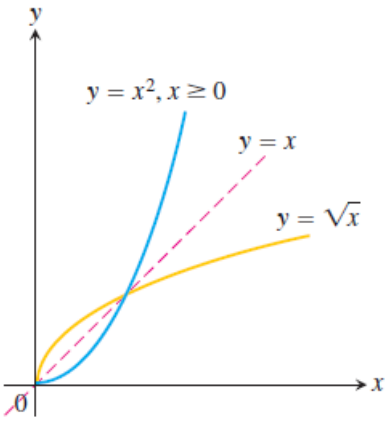
$$\sqrt{y} = \sqrt{x^2} = |x| = x \qquad |x| = x \text{ because } x \geq 0$$

We then interchange x and y, obtaining

$$y = \sqrt{x}.$$

The inverse of the function $y = x^2, x \geq 0$, is the function $y = \sqrt{x}$ (Figure).

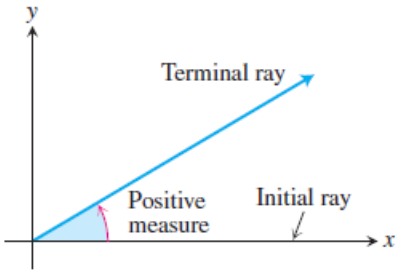
Notice that the function $y = x^2, x \geq 0$, with domain *restricted* to the nonnegative real numbers, *is* one-to-one (Figure) and has an inverse. On the other hand, the function $y = x^2$, with no domain restrictions, *is not* one-to-one (Figure) and therefore has no inverse. ■

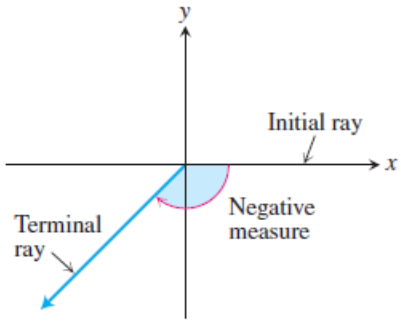


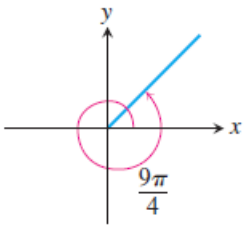
1.9 Trigonometric Functions

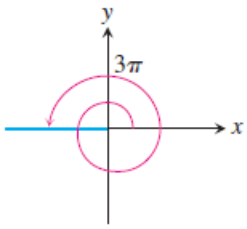
1.9.1 Angles in standard position in the *xy*-plane.

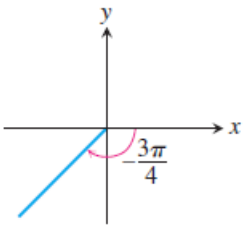
Angles measured counterclockwise from the positive *x*-axis are assigned positive measures; angles measured clockwise are assigned negative measures

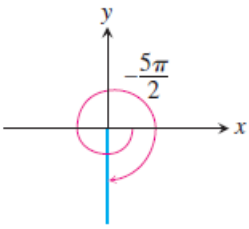




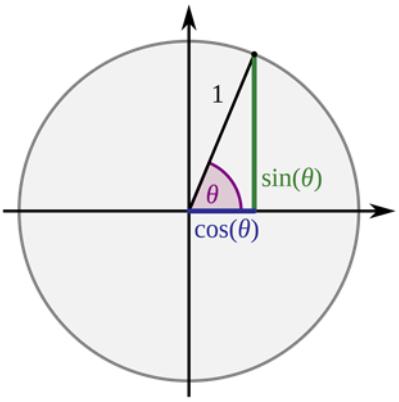


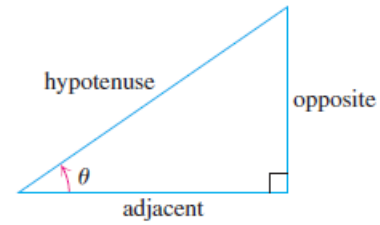






1.9.2 The Six Basic Trigonometric Functions





$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Angle	0	$\pi/2$	π	$3\pi/2$	2π
sinθ	0	1	0	-1	0
Cos θ	1	0	-1	0	1
tanθ	0	+∞	0	-∞	0

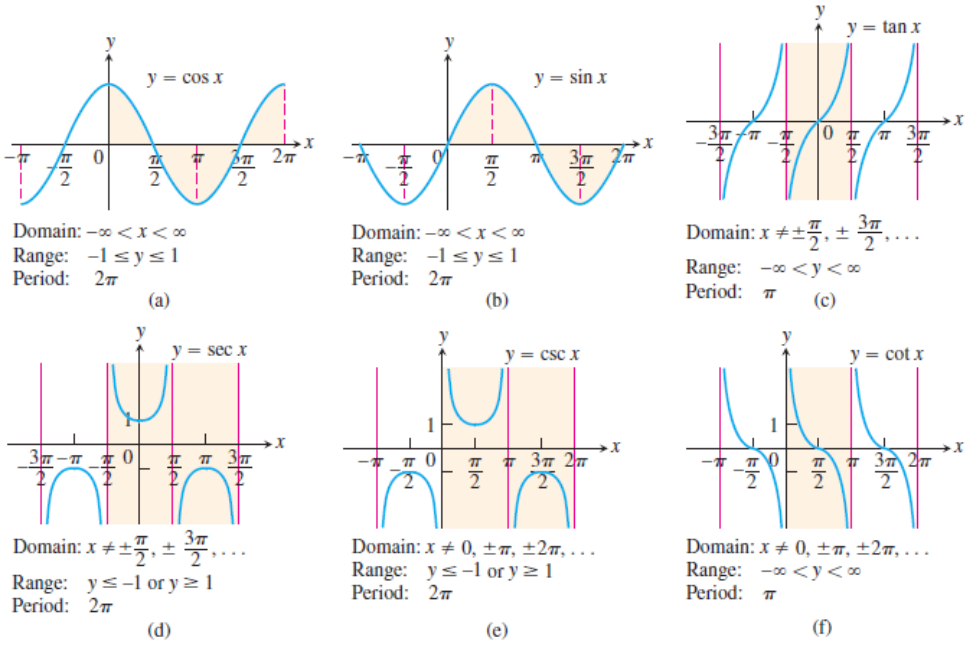


FIGURE Graphs of the six basic trigonometric functions using radian measure. The shading for each trigonometric function indicates its periodicity.

1.9.3 Some of the trigonometric identities

- A.) $\cos^2 x + \sin^2 x = 1$
- B.) $\sin 2x = 2 \sin x \cos x$
- C.) $\cos 2x = 2\cos^2 x - 1$ so that $\cos^2 x = \dfrac{1 + \cos 2x}{2}$
- D.) $\cos 2x = 1 - 2\sin^2 x$ so that $\sin^2 x = \dfrac{1 - \cos 2x}{2}$
- E.) $\cos 2x = \cos^2 x - \sin^2 x$
- F.) $1 + \tan^2 x = \sec^2 x$ so that $\tan^2 x = \sec^2 x - 1$
- G.) $1 + \cot^2 x = \csc^2 x$ so that $\cot^2 x = \csc^2 x - 1$

1.10 Common Functions

Linear Functions A function of the form $f(x) = mx + b$, for constants m and b, is called a **linear function**. Figure below shows an array of lines $f(x) = mx$ where $b = 0$,

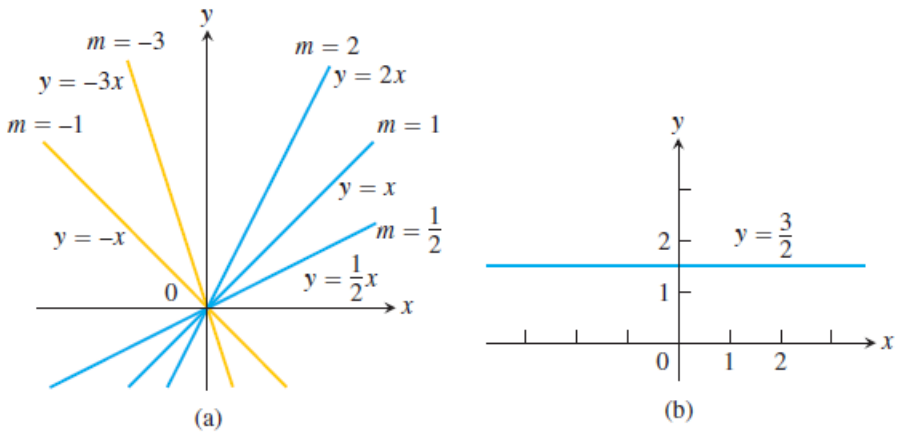
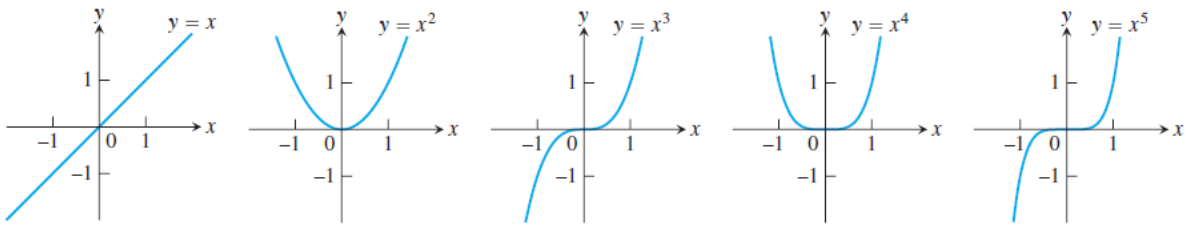
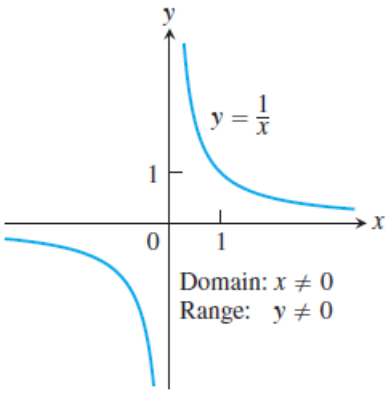


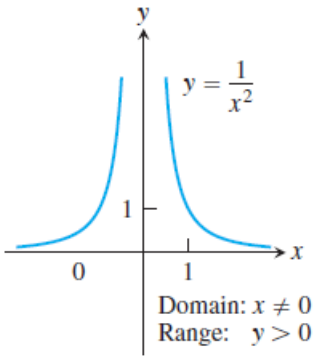
FIGURE (a) Lines through the origin with slope m. (b) A constant function with slope $m = 0$.

Power Functions A function $f(x) = x^a$, where a is a constant, is called a **power function**. There are several important cases to consider.





(a)

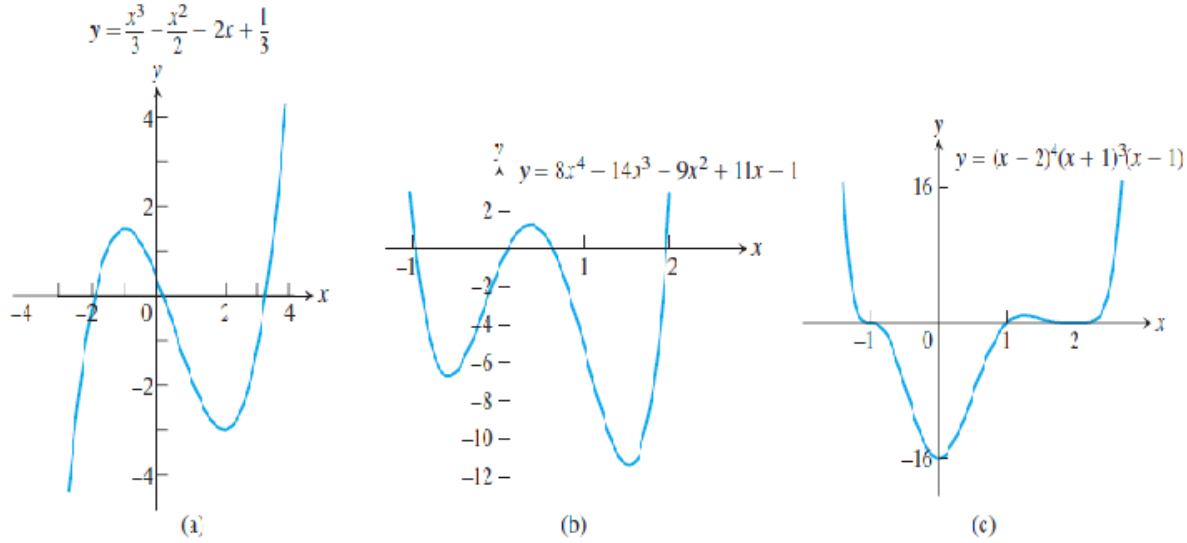


(b)

Polynomials A function p is a **polynomial** if

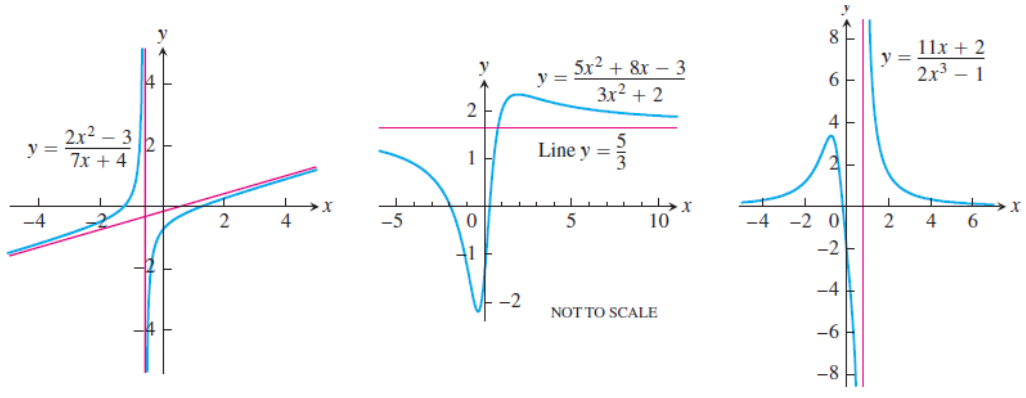
$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots, a_n$ are real constants (called the **coefficients** of the polynomial). All polynomials have domain $(-\infty, \infty)$.



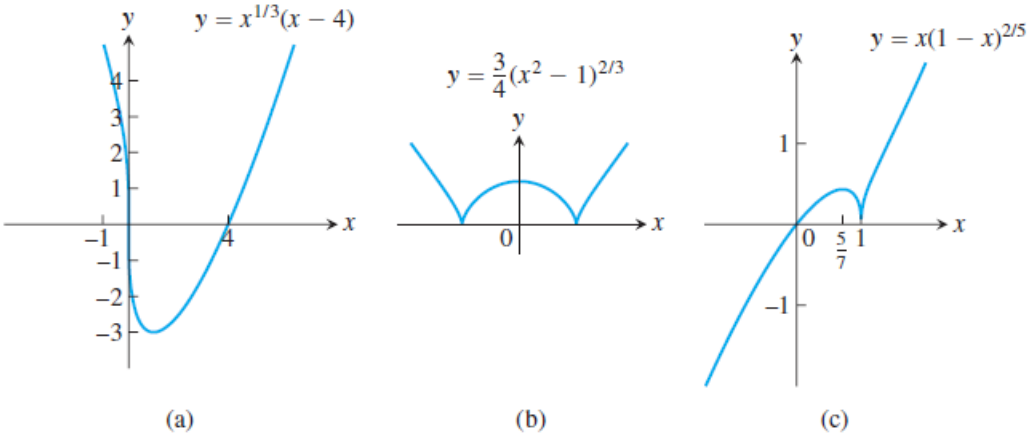
(a) (b) (c)

Rational Functions A **rational function** is a quotient or ratio $f(x) = p(x)/q(x)$, where p and q are polynomials. The domain of a rational function is the set of all real x for which $q(x) \neq 0$. The graphs of several rational functions are shown below



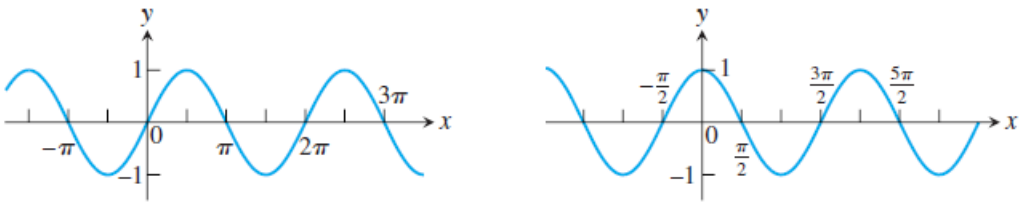
(a) (b) (c)

Algebraic Functions Any function constructed from polynomials using algebraic operations (addition, subtraction, multiplication, division, and taking roots) lies within the class of **algebraic functions**.



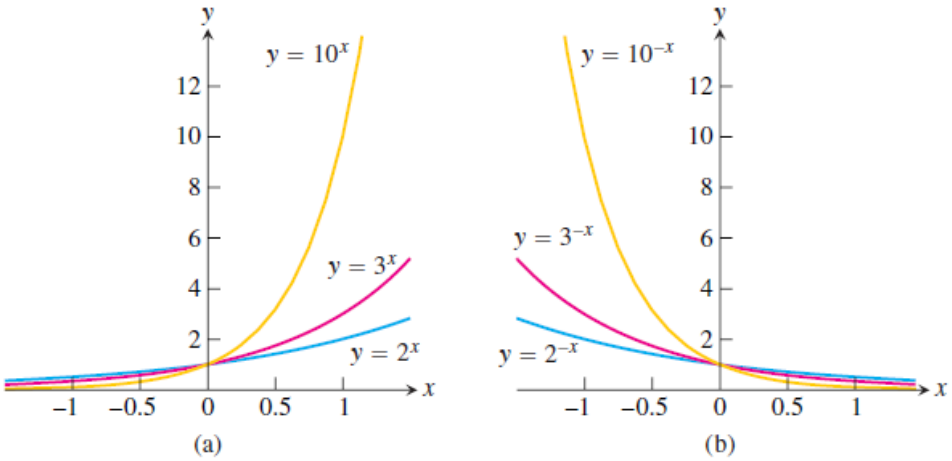
(a) (b) (c)

Trigonometric Functions The six basic trigonometric functions are reviewed before and The graphs of the sine and cosine functions are shown below

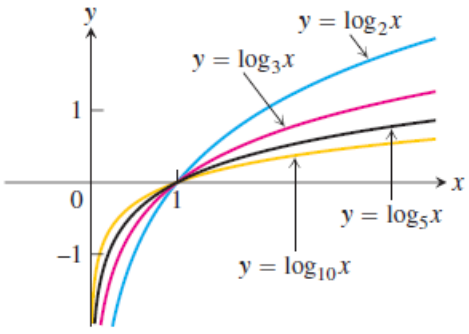


(a) $f(x) = \sin x$ (b) $f(x) = \cos x$

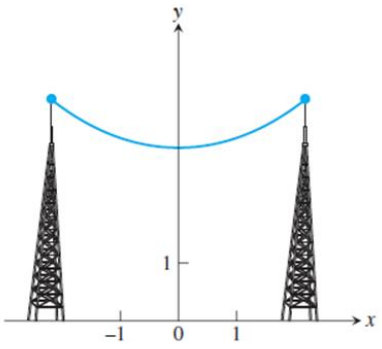
Exponential Functions Functions of the form $f(x) = a^x$, where the base $a > 0$ is a positive constant and $a \neq 1$, are called **exponential functions**. All exponential functions have domain $(-\infty, \infty)$ and range $(0, \infty)$, so an exponential function never assumes the value 0.



Logarithmic Functions These are the functions $f(x) = \log_a x$, where the base $a \neq 1$ is a positive constant. They are the *inverse functions* of the exponential functions



Transcendental Functions These are functions that are not algebraic. They include the trigonometric, inverse trigonometric, exponential, and logarithmic functions, and many other functions as well. A particular example of a transcendental function is a **catenary**. Its graph has the shape of a cable, like a telephone line or electric cable, strung from one support to another and hanging freely under its own weight (Figure).



1.10.1 Recognizing Functions

1. a. $f(x) = 7 - 3x$ **b.** $g(x) = \sqrt[5]{x}$

c. $h(x) = \dfrac{x^2 - 1}{x^2 + 1}$ **d.** $r(x) = 8^x$

2. a. $F(t) = t^4 - t$ **b.** $G(t) = 5^t$

c. $H(z) = \sqrt{z^3 + 1}$ **d.** $R(z) = \sqrt[3]{z^7}$

3. a. $y = \dfrac{3 + 2x}{x - 1}$ **b.** $y = x^{5/2} - 2x + 1$

c. $y = \tan \pi x$ **d.** $y = \log_7 x$

4. a. $y = \log_5\left(\dfrac{1}{t}\right)$ **b.** $f(z) = \dfrac{z^5}{\sqrt{z} + 1}$

c. $g(x) = 2^{1/x}$ **d.** $w = 5 \cos\left(\dfrac{t}{2} + \dfrac{\pi}{6}\right)$

Answer

1. (a) linear, polynomial of degree 1, algebraic. (b) power, algebraic.
   (c) rational, algebraic. (d) exponential.

2. (a) polynomial of degree 4, algebraic. (b) exponential.
   (c) algebraic. (d) power, algebraic.

3. (a) rational, algebraic. (b) algebraic.
   (c) trigonometric. (d) logarithmic.

4. (a) logarithmic. (b) algebraic.
   (c) exponential. (d) trigonometric.

Questions

1.In Ex 1-4 find the domain and range of each function also find the type of the function

1. $f(x) = 1 + x^2$ **2.** $f(x) = 1 - \sqrt{x}$

3. $F(x) = \sqrt{5x + 10}$ **4.** $g(x) = \sqrt{x^2 - 3x}$

Piecewise-Defined Functions

2. Graph the function

. $G(x) = \begin{cases} 1/x, & x < 0 \\ x, & 0 \le x \end{cases}$

3.Increasing and Decreasing Functions

Graph the functions in the following Exercises and find their type. What symmetries, if any, do the graphs have? Specify the intervals over which the function is increasing and the intervals where it is decreasing.

1.$y = -x^3$ 2. $y = -\frac{1}{x}$ 3.$y = \sqrt{x}$

4. $y = f(x) = \begin{cases} -x, & x < 0 \\ x, & x \ge 0 \end{cases}$ 5.

4. Even and odd functions

In Exercises 47–58, say whether the function is even, odd, or neither. Give reasons for your answer.

47. $f(x) = 3$

48. $f(x) = x^{-5}$

49. $f(x) = x^2 + 1$

50. $f(x) = x^2 + x$

51. $g(x) = x^3 + x$

52. $g(x) = x^4 + 3x^2 - 1$

53. $g(x) = \dfrac{1}{x^2 - 1}$

54. $g(x) = \dfrac{x}{x^2 - 1}$

5. Composite functions

In Exercises 17 and 18, **(a)** write formulas for $f \circ g$ and $g \circ f$ and find the **(b)** domain and **(c)** range of each.

17. $f(x) = \sqrt{x + 1},\ g(x) = \dfrac{1}{x}$

18. $f(x) = x^2,\ g(x) = 1 - \sqrt{x}$