2nd Year, Springs, Strength of Materials

A spring is a device used to absorb or store energy and release it when required.

The two types of springs are:

1- laminated or leaf springs.

2- Helical springs.

* laminated or leaf spring.

Fig.(1) shows a laminated spring which consists of a number of parallel strips of metal having different lengths and same width placed one over the other.

When the spring is loaded to the designed load W, all the plates becomes flat and the central deflection (δ) disappears.

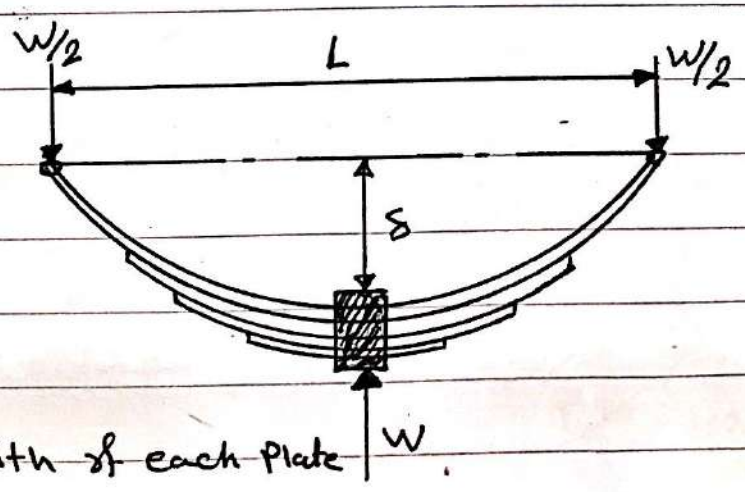


Fig.(1).

Let

b = width of each plate

n = Number of plates

L = Span of spring

σ_b = max. bending stress

t = thickness of each plates.

W = Point load acting at the center of the spring.
δ = original deflection of the top spring.

bending at the centre = load at one end × $\frac{L}{2}$

* $M = \frac{W}{2} \times \frac{L}{2} = \frac{WL}{4}$ ---- ①

=point load x spam of spring /4

The Moment of inertia of each Plate,

$I = \frac{b t^3}{12}$

= width of each plate x thickness of each plate 3 /12

$M = \frac{\sigma_b}{y} \times I$ (here $y = \frac{t}{2}$)

=maximum bending stress x moment of inertia x 2/ thickness of each plate

$M = \frac{\sigma_b \times b t^3/12}{t/2} = \frac{\sigma_b \times b \times t^2}{6}$

* total resisting Moment by n Plates

$M = n \times m$

$= \frac{n \sigma_b \times b \times t^2}{6}$ ---- ②

equating ① and ②

$\frac{WL}{4} = \frac{n \sigma_b \times b \times t^2}{6}$

* $\sigma_b = \frac{3WL}{2nbt^2}$ ---- ③

maximum bending stress

Now R = radius of the plate to which they are bent.

From triangle ACO

$(AO)^2 = (AC)^2 + (CO)^2$

or $R^2 = (\frac{L}{2})^2 + (R-\delta)^2$

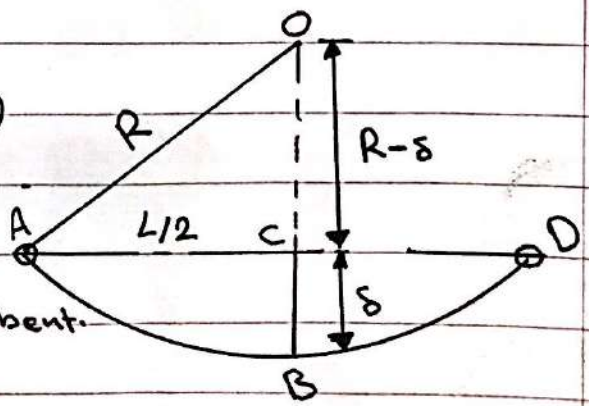


Fig.(2).

$$R^2 = \frac{L^2}{4} + R^2 + \delta^2 - 2R\delta$$

(Neglecting δ^2 which is a small quantity)

$$\therefore 2R\delta = \frac{L^2}{4}$$

$$* \quad \delta = \frac{L^2}{4*2R} = \frac{L^2}{8R} \quad \text{---- (4)}$$

But the relation between bending stress, Modulus of elasticity and radius of curvature (R) is given by

$$\frac{\sigma_b}{y} = \frac{E}{R}$$

$$R = \frac{E*y}{\sigma_b} = \frac{E*t}{2\sigma_b} \qquad (\text{here } y = \frac{t}{2})$$

Substituting this value of R in eq (4) we get

$$* \quad \delta = \frac{L^2*2\sigma_b}{8Et} = \frac{\sigma_b \times L^2}{4Et} \quad \text{---- (5)}$$

ex1/ A leaf spring carries a central load of 3000N. The leaf spring is to be made of 10 steel plates 5 cm wide and 6 mm thick. If the bending stress is limited to 150 N/mm^2 determine:

(a) length of the spring.

(b) deflection at the center of the spring.

take $E = 2*10^5$ N/mm^2.

Sol.

(a) $\sigma_b = \frac{3WL}{2nbt^2}$ equation 3 page 2

$$150 = \frac{3*300*L}{2*10*50*6^2}$$

$\therefore L = 600\,mm$

(b)

$$\delta = \frac{\sigma_b \cdot L^2}{4Et} = \frac{150*600^2}{4*2*10^5*6} = 11.25\,mm$$ equation 5 page 3

* Helical Springs - Helical spring are the thick spring wires coiled into a helix. They are of two types:

1 - Close-coiled helical springs.

2 - open-coiled helical springs.

* Close-coiled helical springs, are the spring in which helix angle is very small or in other words the pitch between two adjacent turns is small. as shown in fig. (3). in this case the bending effect on the spring is ignored and we assume that the close-coiled helical springs are to stand purely torsional stresses.

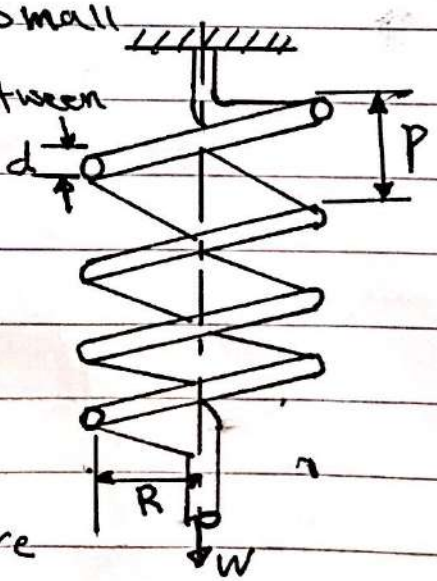


fig (3)

Let d = Diameter of spring wire

p = Pitch of the helical spring

n = Number of coils

R = mean radius of spring coil

W = Axial load

G = Modulus of rigidity

τ = Max. shear stress induced in wire.

θ = angle of twist in spring wire

δ = deflection of spring due to axial load.

L = length of wire.

Now twisting moment on the wire

$$T = W * R \quad \text{---} (1)$$

also given by, $T = \frac{\pi}{16} \tau d^3$ ---- (2)

(Equation 2a page 4 of torsional formula in the last lecture of torsion)
d is the diameter of spring wire.

Equating (1) and (2)

$$W * R = \frac{\pi}{16} \tau d^3$$

* $\tau = \frac{16 W * R}{\pi d^3}$ 6

maximum shear stress
w= axial load, R= mean radius of spring coil, d= diameter of spring wire.

* length of the coil $= \pi D$ or $2\pi R$ 7

Total length of the wire $= 2\pi R * n$ n = mumber of coils

* $\delta = \frac{64 W R^3 n}{G d^4}$ (δ = deflection) 8

G= Modulus of rigidity

* The stiffness of spring (S)

S = load Per unit deflection

* $S = \frac{W}{\delta} = \frac{W}{\frac{64 W R^3 n}{G d^4}} = \frac{G d^4}{64 R^3 n}$ 9

Note The solid length of the spring means the distance between the coils when the coils are touching each other. There is no gap between the coils. The solid length is given by

* **Solid Length = nd** n= number of coils, d=diameter of spring wire 10

ex2/ A closely coiled helical spring is to carry a load of 500N. Its mean coil diameter is to be 10 times that of the wire diameter. Calculate these diameters if the max. shear stress in the material of the spring is to be $80\ N/mm^2$.

Sol.

$D = 10d$

$$\tau = \frac{16WR}{\pi d^3}$$ equation 6 page 5

$$80 = \frac{16 * 500 * (D/2)}{\pi d^3} \qquad (\because R = \frac{D}{2})$$

$$80 = \frac{8000 * (\frac{10d}{2})}{\pi d^3} \qquad (D = 10d)$$

$$80 * \pi d^3 = 8000 * 5d$$

$$d = 12.6\ mm$$

$$\therefore D = 10 * d = 10 * 12.6 = 126\ mm.$$

ex3/ in ex2, if the stiffness of the spring is 20N per mm deflection and modulus of rigidity $= 8 * 10^4\ N/mm^2$, find the number of coils in the closely helical spring.

Sol.

$$S = \frac{\text{load}}{\delta}$$ equation9 page 5

$$20 = \frac{500}{\delta} \Rightarrow \delta = \frac{500}{20} = 25\,mm$$

$$\delta = \frac{64 W R^3 n}{G d^4}$$ equation 8 page 5

$$25 = \frac{64 \times 500 \times (63)^3 \times n}{8.4 \times 10^4 \times (12.6)^4} \qquad (R = 63mm)\ (cms)$$

$$n = \frac{25 \times 8.4 \times 10^4 \times 12.6^4}{64 \times 500 \times (63)^3} = 6.6 \text{ say } 7$$

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ex4/ A closely coiled helical spring of round steel wire 10mm in diameter having 10 complete turns with a mean diameter of 120mm is subjected to an axial load of 200 N. Determine (i) the deflection of the spring (ii) max. shear stress in the wire, (iii) stiffness of the spring, Take $G = 8 \times 10^4\ N/mm^2$.

Sol.

(i) $$\delta = \frac{64 W R^3 n}{G d^4} = \frac{64 \times 200 \times 60^3 \times 10}{8 \times 10^4 \times 10^4} = 34.5\,mm$$ equation 8 page 5

(ii) $$\tau = \frac{16 W R}{\pi d^3} = \frac{16 \times 200 \times 60}{\pi \times 10^3} = 61.1\ N/mm^2$$ equation 6 page 5

(iii) $$S = \frac{W}{\delta} = \frac{200}{34.5} = 5.8\ N/mm$$ equation 9 page 5

Good luck