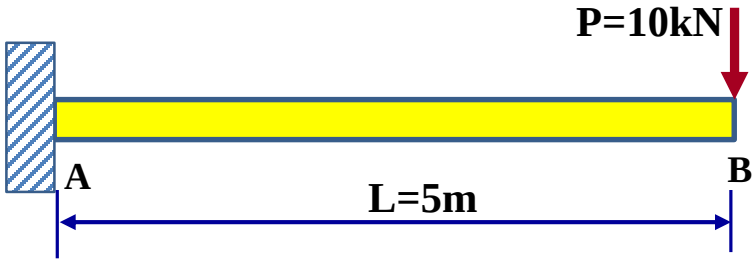
ASHUR UNIVERSITY

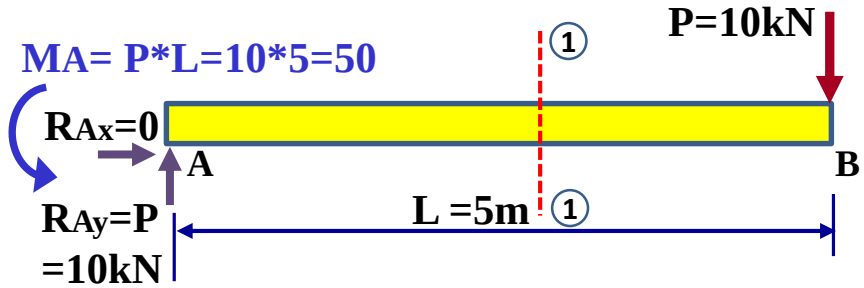
Chapter-2

Part - 2

Axial Force, Shear Force and Bending moment

Example 2-4: Draw axial, shear force and bending moment diagram of Cantilever Beam AB carrying concentrated load P = 10 kN at free end, shown below:

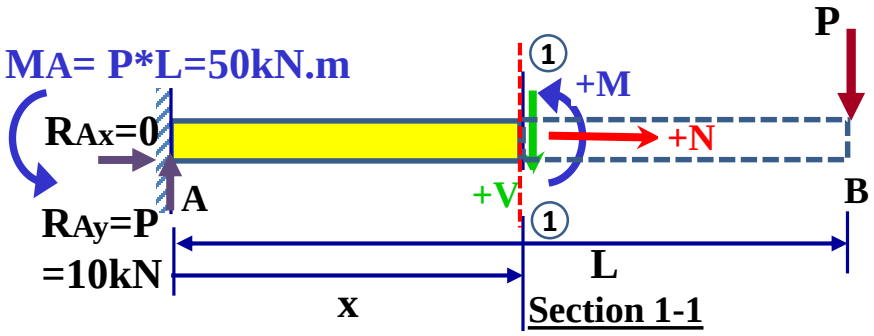




Solution: step 1:

Σ Fx = 0 ➡ RAx = 0 and ΣMA = 0 ✤ , MA - P*L = 0, MA = P*L = 10*5 = 50kN-m

ΣFy = 0, ➡ RAy – P= 0 ➡ RAy = P = 10kN



step 2: Define elements

Part AB (0 ≤ x ≤ 5m)

Load	x = 0	0 < x < 5m	x = 5m
N	0	0	0
V	+ 10	+ 10	+10
M	- 50	+ 10*x – 50	0

Sec. 1-1 :

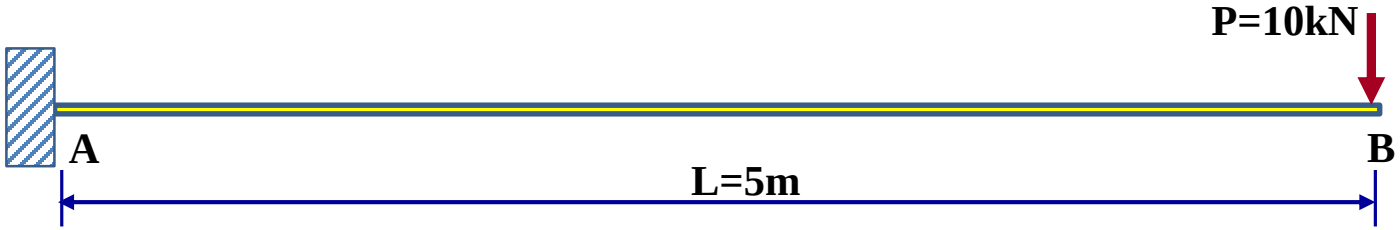
Σ Fx = 0 ✤ RAx + N= 0, 0 + N = 0, N=0

Σ Fy = 0, ✤ RAy – V= 0, 10 – V=0, V = +10kN

Σ Mz = 0 , ✤ M + MA- RAy*x= 0, M +50- 10x=0,

M = +10x - 50

step 3: Draw axial force (N), Shear force (V) and Moment (M) Diagram along beam AB

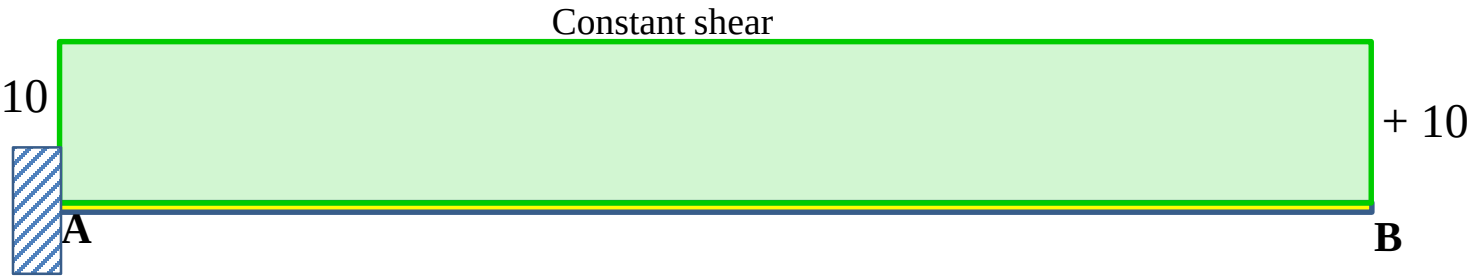


Axial Force Diagram (A.F.D.)

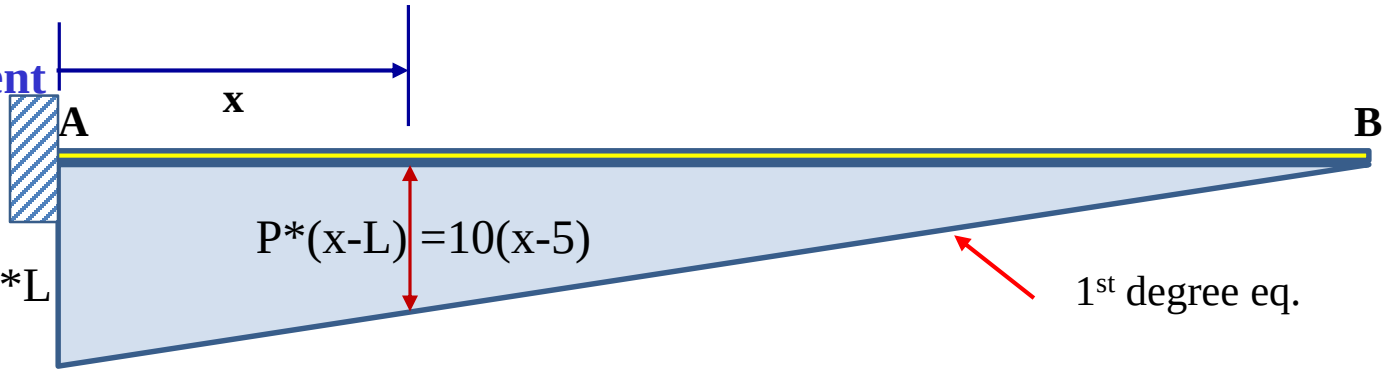


N= 0

Shear Force Diagram (S.F.D.)

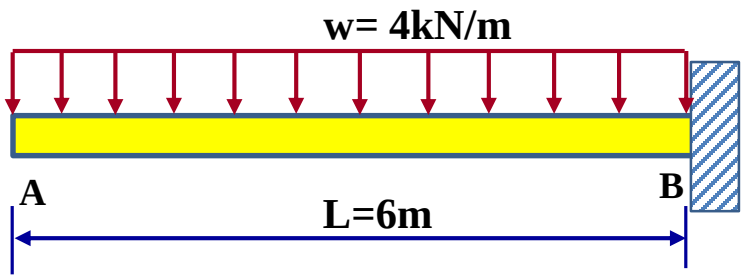


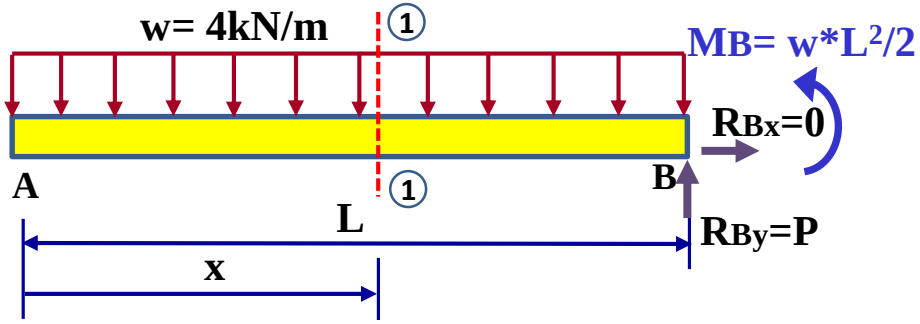
Bending Moment Diagram (B.M.D.)



Mmax=- P*L
=-50

Example 2-5: Draw axial force, shear force and bending moment diagram of Cantilever Beam AB carrying a uniformly full distributed load w, shown below:





Solution: step 1:

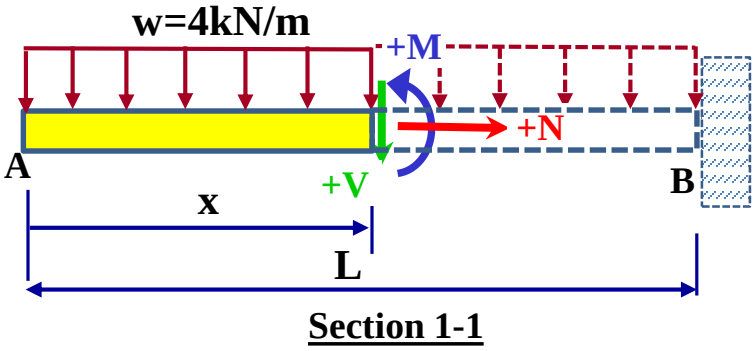
$\Sigma F_x = 0 \Rightarrow R_{Bx} = 0$ and $\Sigma M_B = 0$, $M_B + (w*L)*L/2 = 0, \Rightarrow M_B = -w*L^2/2 = -4*6^2/2 = -72$kN-m

$\Sigma F_y = 0, \Rightarrow R_{By} - w*L = 0 \Rightarrow R_{By} = w*L = 4 * 6 = 24$kN

step 2: Define elements

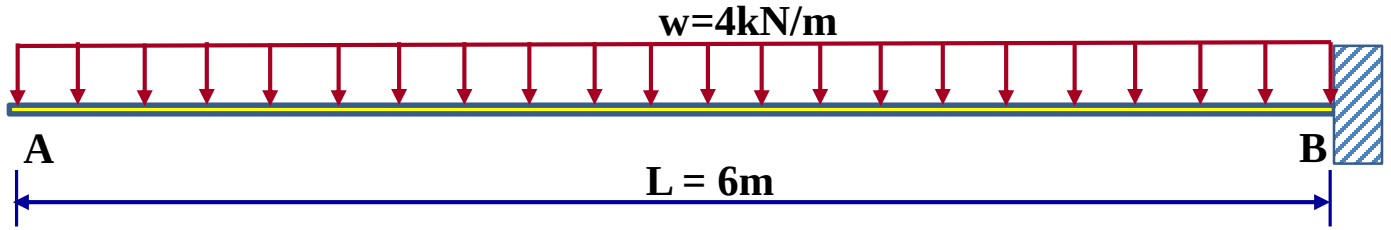
Part AB ($0 \le x \le 6$m)

Load	x = 0	0 < x < 6	x = 6
N(kN)=	0	0	0
V(kN)=	0	- 4*x	- 4*L
M	0	- 4*x*(x/2)	- w*L2/2=-72



Section 1-1

step 3: Draw axial force (N), Shear force (V) and Moment (M) Diagram along beam AB

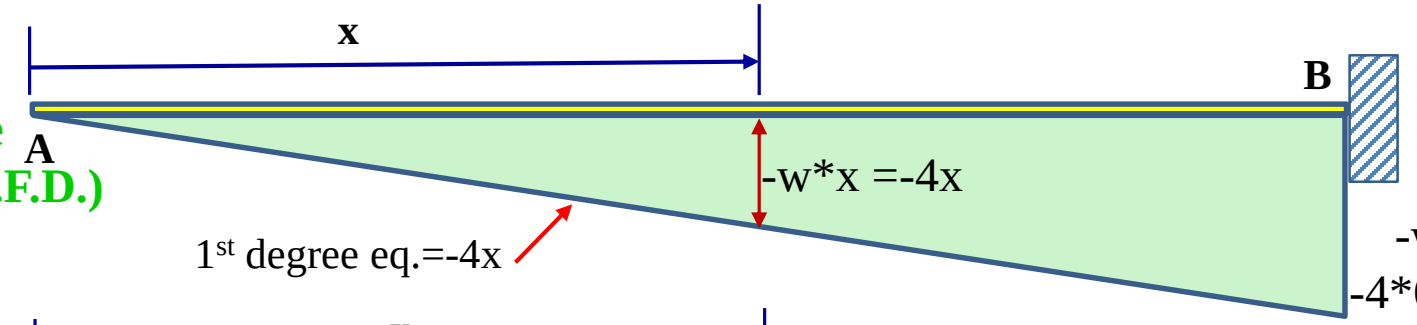


Axial Force Diagram(A.F.D.)

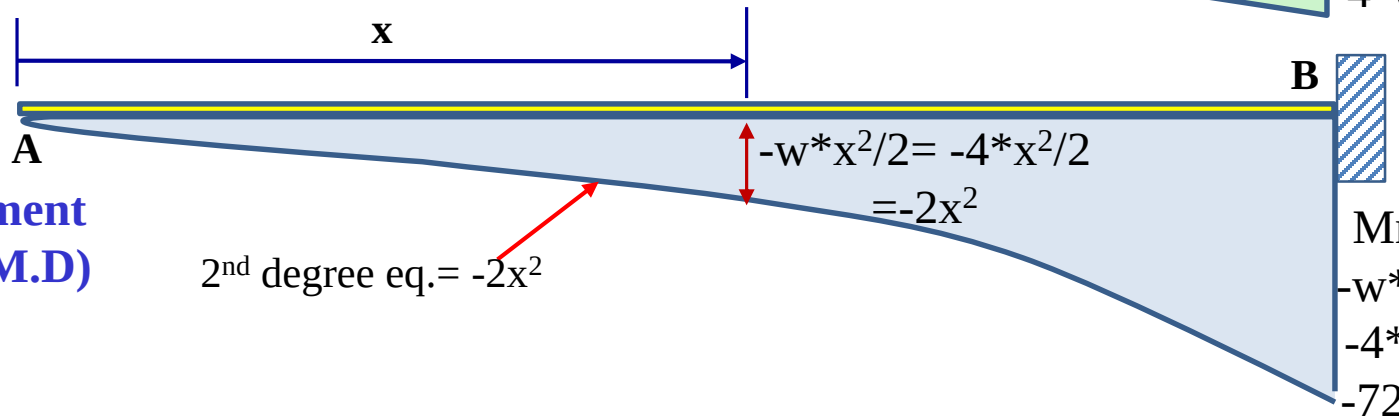
N= 0



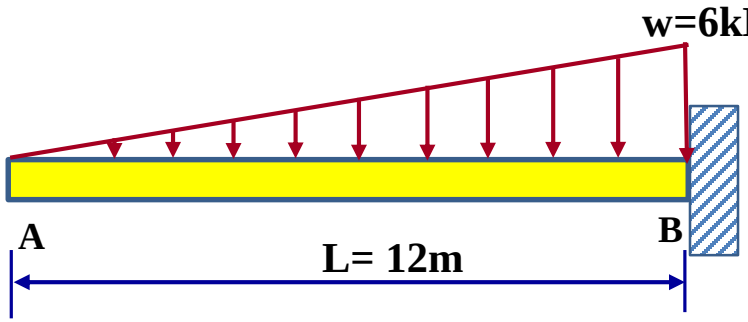
Shear Force Diagram (S.F.D.)



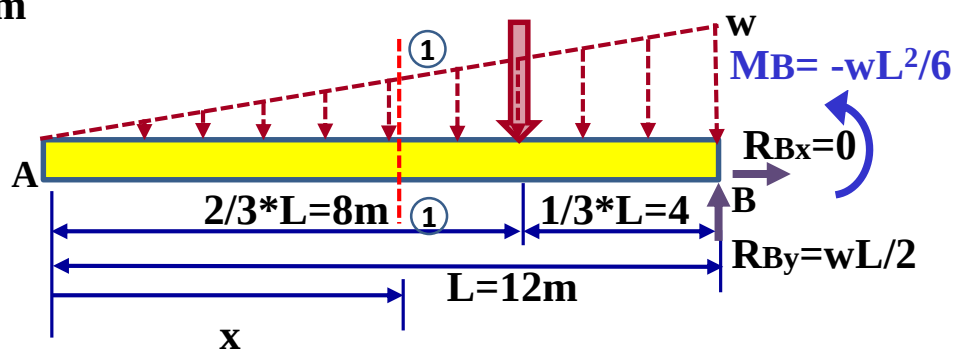
Bending Moment Diagram (B.M.D)



Example 2-6: Plot axial force, shear force and bending moment diagram of Cantilever Beam AB carrying a load increasing uniformly to one end w, shown below:



Equivalent load W=w*L/2=6*12/2=36kN



Solution: step 1:

$\Sigma F_x = 0 \Rightarrow R_{Bx} = 0$

$\Sigma M_B = 0, \Rightarrow M_B + (W)*L/3 = 0, \Rightarrow M_B + (w*L/2)*L/3 = 0 \Rightarrow M_B = -w*L^2/6 = -6*12^2/6 = -144 kN\text{-}m$

$\Sigma F_y = 0, \Rightarrow R_{By} - w*L/2 = 0 \Rightarrow R_{By} = w*L/2 = 6*12/2 = 36kN$

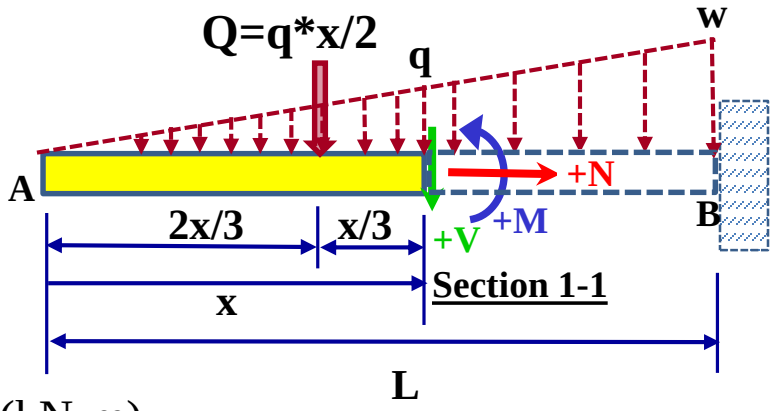
By triangle sides relationship:

$$\frac{q}{x} = \frac{w}{L} \Rightarrow q = \frac{w}{L}*x = \frac{6}{12}*x = 0.5x$$

Equivalent load of triangle load span x:

$Q = q*x/2 \Rightarrow V(x) + Q = 0 \Rightarrow V(x) = -Q$

$$\therefore V(x) = -Q = -\frac{w*x^2}{2L} = -\frac{6*x^2}{2*12} = -0.25x^2\ (kN)$$

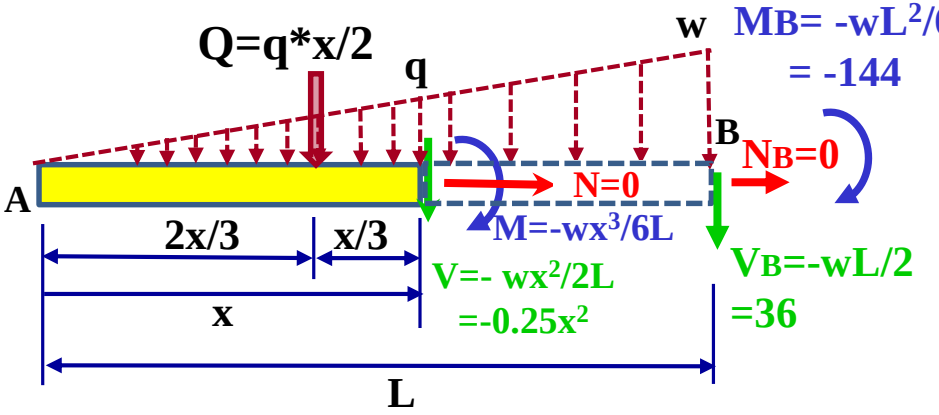


and $M(x) = -Q*x/3 = -\frac{w*x^3}{6L} = -\frac{6*x^3}{6*12} = -0.08333x^3 (kN\text{-}m)$

step 2: Define elements

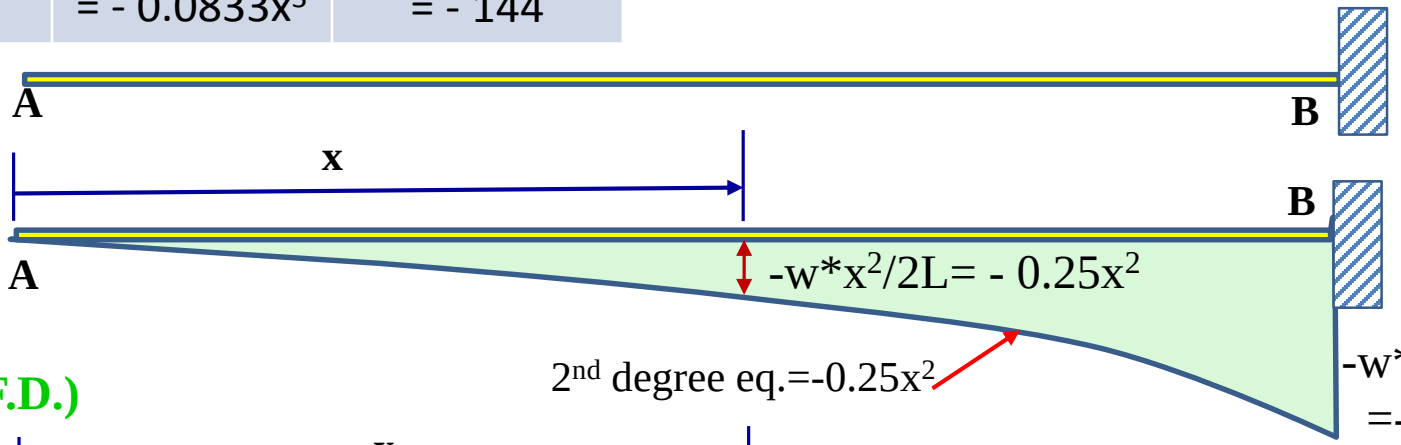
Part AB (0 ≤ x ≤ 12m)

Load	x = 0	0 < x < 12	x = 12
N(kN)	0	0	0
V(kN)	0	$- w*x^2/2L$ $= -0.25x^2$	$- w*L/2$ $= - 36$
M(kn-m)	0	$- w*x^3/6L$ $= - 0.0833x^3$	$- w*L^2/6$ $= - 144$

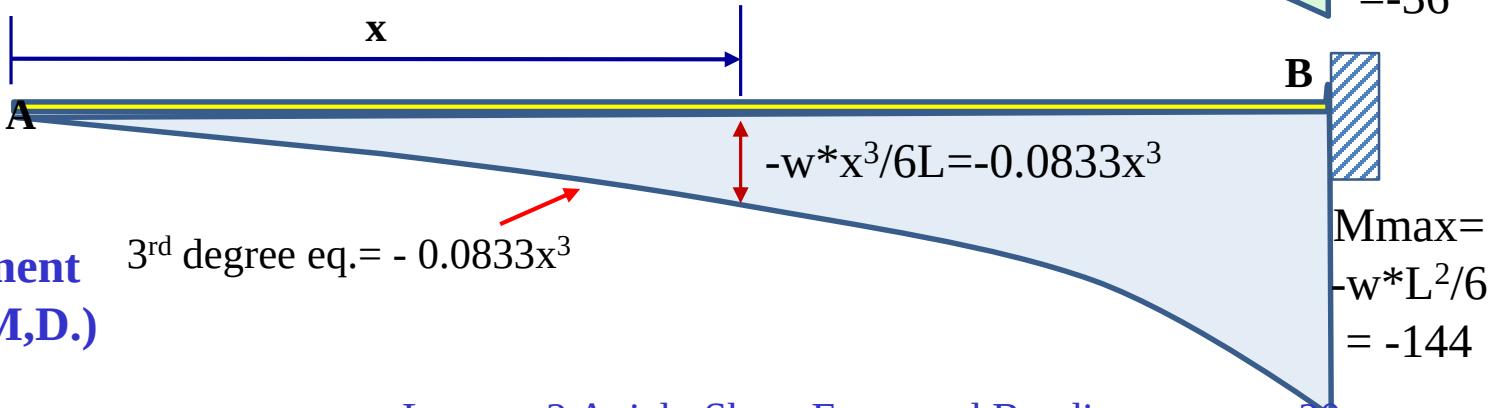


Axial Force Diagram

N= 0(A.F.D.)

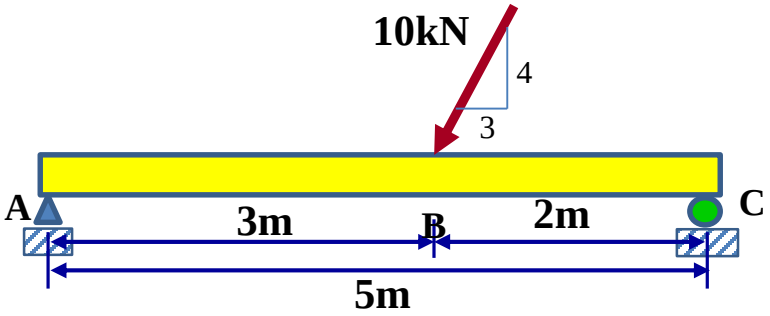


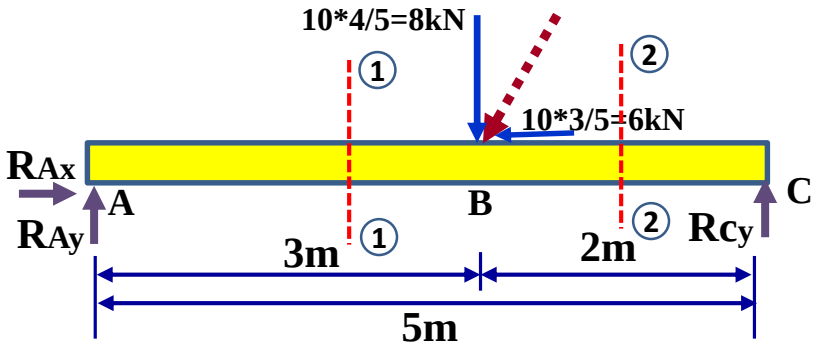
Shear Force Diagram (S.F.D.)



Bending Moment Diagram (B.M,D.)

Example 2-7: Draw axial force, shear force and bending moment diagram of simply supported beam ABC carrying inclined concentrated load 10kN, shown below:





Solution: step 1:

$\Sigma F_x = 0$ → $R_{Ax} - 6 = 0$, $R_{Ax} = 6$ →

$\Sigma M_c = 0$, $-R_{Ay} * 5 + 8 * 2 = 0$, $R_{Ay} = 16/5 = 3.2kN$

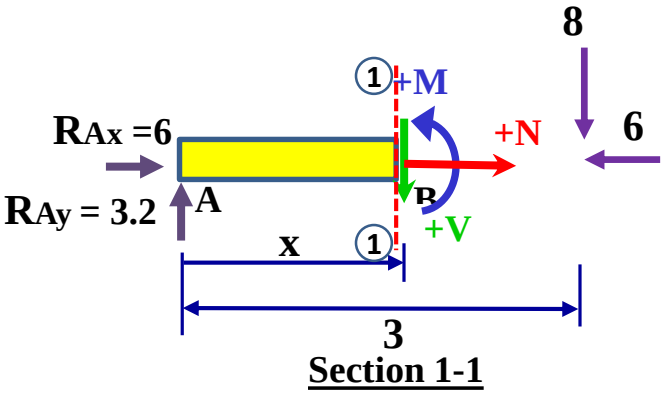
$\Sigma F_y = 0$ ↑ , $R_{Ay} + R_{cy} - P = 0$, $3.2 + R_{cy} - 8 = 0$, $R_{cy} = 8 - 3.2 = 4.8kN$ ↑

step 2: Define elements: sec. 1-1 Part AB ($0 \le x \le 3$)

$\Sigma F_x = 0$ → $R_{Ax} + N = 0$, $6 + N = 0$, **N = -6**

$\Sigma F_y = 0$ ↑ $R_{Ay} - V = 0$, $3.2 - V = 0$, **V = + 3.2**

$\Sigma M_z = 0$ $M - 3.2 * x = 0$, **M = + 3.2x**



Load	x = 0	0 < x < 3	x = 3
N(kN)=	-6	-6	-6
V(kN)=	+ 3.2	+ 3.2	+3.2
M(kN-m)=	0	+ 3.2*x	+ 9.6

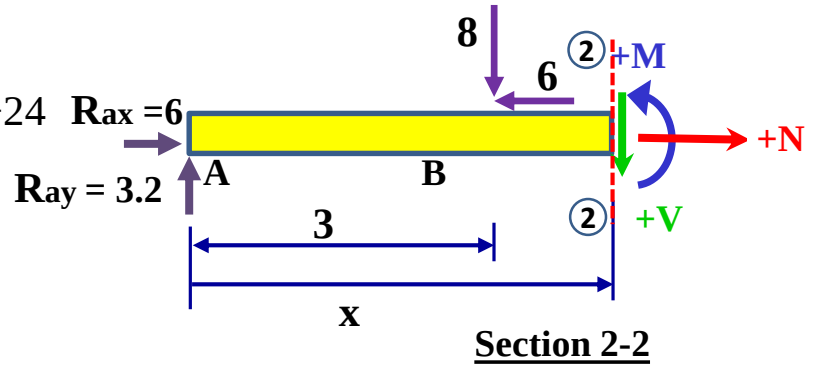
Sec/ 2-2 :Part BC (3 ≤ x ≤ 5)

Σ Fx = 0 ⊕→ R_{Ax} + N – 6 = 0 , 6 + N - 6 = 0, **N = 0**

Σ Fy = 0 ⊕↑ R_{Ay} – 8 - V = 0 , 3.2 - 8 - V = 0, **V = - 4.8**

Σ Mz = 0 ⊕↺ M – 3.2 *x + 8*(x-3) = 0, M = + 3.2x – 8*x +24

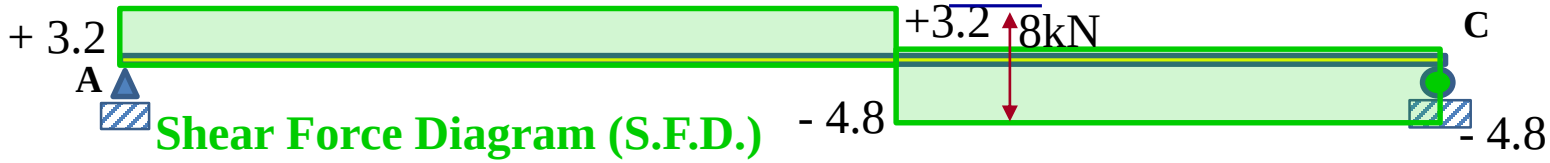
M = 24 -4.8x



Load	x = 3	3 < x < 5	x = 5
N(kN)=	0	0	0
V(kN)=	- 4.8	- 4.8	- 4.8
M(kN-m)=	+ 9.6	24-4.8*x	0

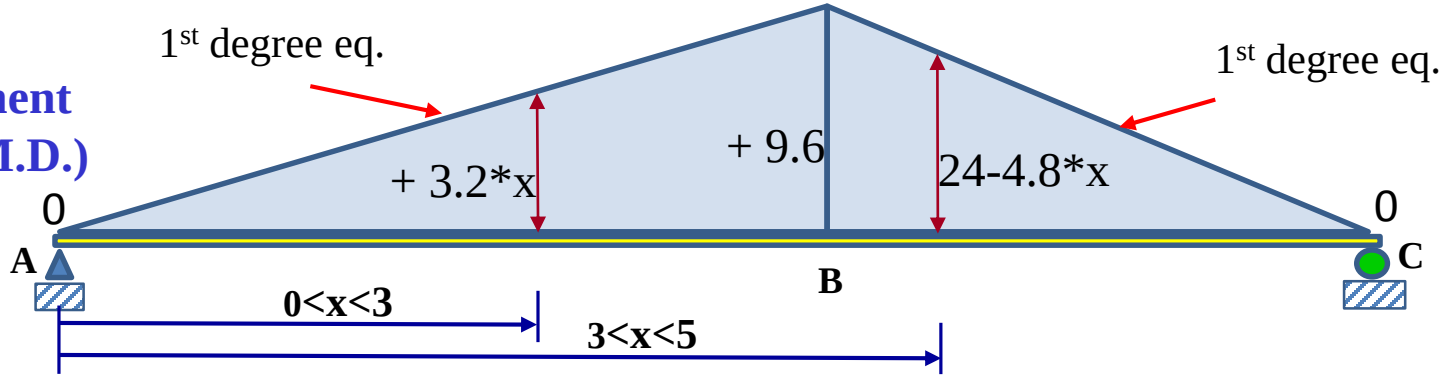
Axial Force Diagram(A.F.D.)



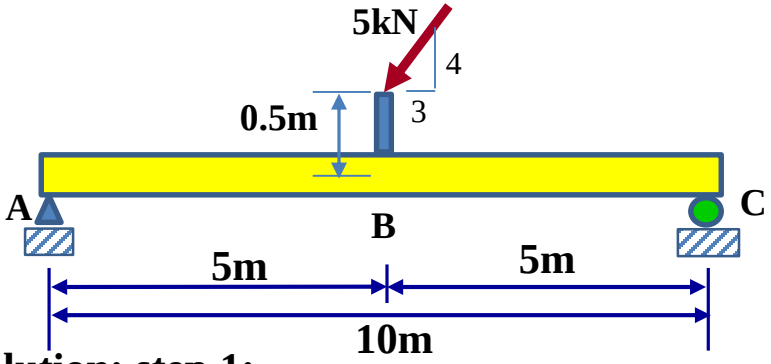


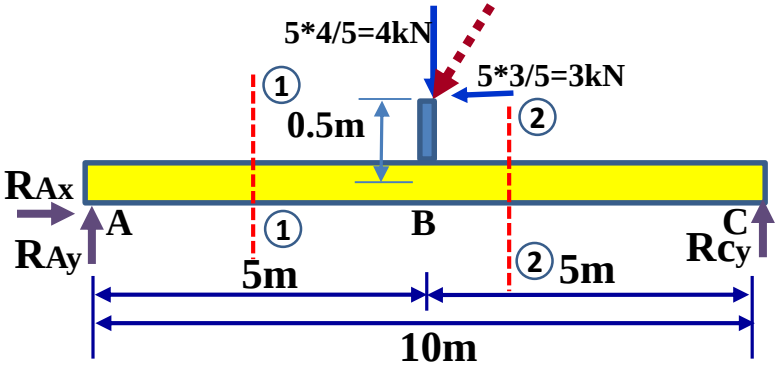
Shear Force Diagram (S.F.D.)

Bending Moment Diagram (B.M.D.)



Example 2-8: Draw axial force, shear force and bending moment diagram of simply supported beam ABC carrying inclined concentrated load 5kN, shown below:





Solution: step 1:

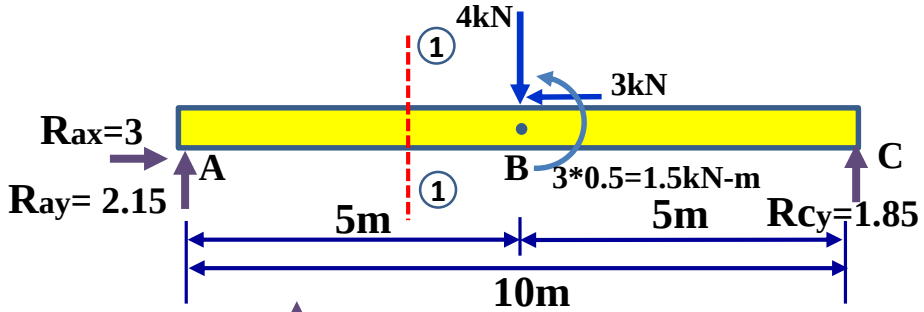
$\Sigma F_x = 0 \rightarrow \quad R_{Ax} - 3 = 0, \quad R_{Ax} = 3 \rightarrow$

$\Sigma \quad M_A = 0 \quad \circlearrowleft, \quad -4*5 + R_{Cy} * 10 + 1.5 = 0,$

$\Sigma \quad R_{Cy} = 18.5/10 = 1.85kN \uparrow$

$\Sigma \quad F_y = 0 \uparrow \quad, \quad R_{Ay} + R_{Cy} - 4 = 0$

$R_{Ay} + 1.85 - 4 = 0, \quad R_{Ay} = 4 - 1.85 = 2.15kN \uparrow$



step 2: Define elements

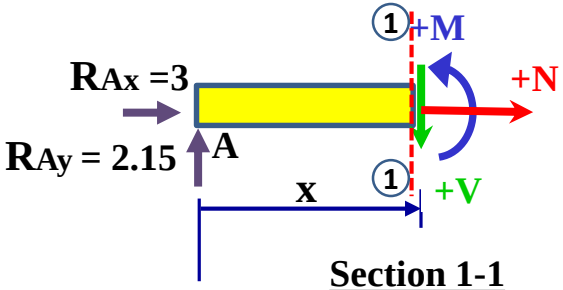
Sec.1-1: Part AB ($0 \le x \le 5$)

$\Sigma F_x = 0 \rightarrow \quad R_{Ax} + N = 0, \quad 3 + N = 0, \quad$ **N = - 3**

$\Sigma \quad F_y = 0 \uparrow \quad, \quad R_{Ay} - V = 0$

$2.15 - V = 0, \quad$ **V = 2.15**

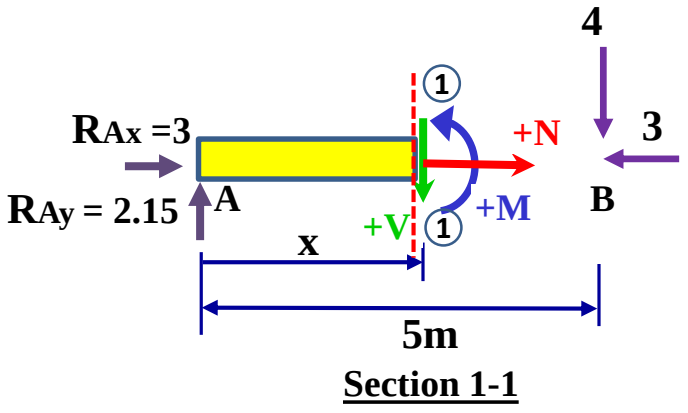
$\Sigma M_z = 0 \quad \circlearrowleft, \quad -2.15*x + M_z = 0, \quad$ **Mz = 2.15x**



Section 1-1

Part AB (0 ≤ x ≤ 5)

Load	x = 0	0 < x < 5	x = 5
N(kN)=	-3	-3	-3
V(kN)=	+ 2.15	+ 2.15	+ 2.15
M(kN-m)=	0	+ 2.15*x	+ 10.75



Section 1-1

Sec.2-2: Part BC (5≤ x ≤ 10)

$\Sigma F_x = 0$ → $R_{Ax} - 3 + N = 0$, $3 - 3 + N = 0$, **$N = 0$**

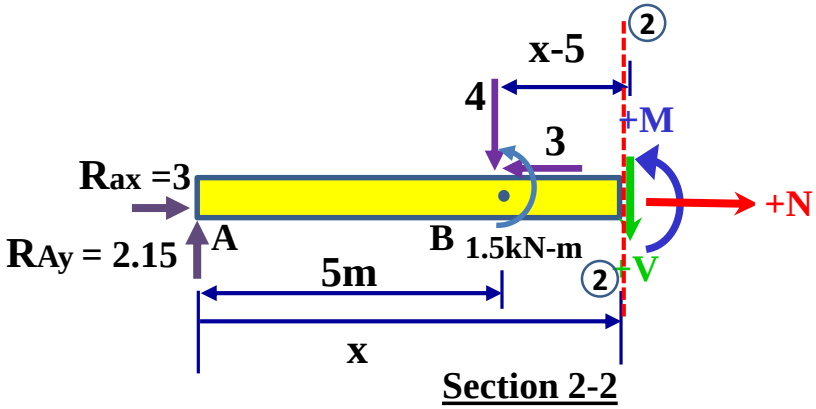
$\Sigma F_y = 0$ ↑ , $R_{Ay} - 4 - V = 0$

$2.15 - 4 - V = 0$, **$V = -1.85$**

$\Sigma M_z = 0$, $-2.15*x + 1.5 + 4*(x-5) + M_z = 0$,

$M_z = 2.15x - 1.5 - 4x + 20$

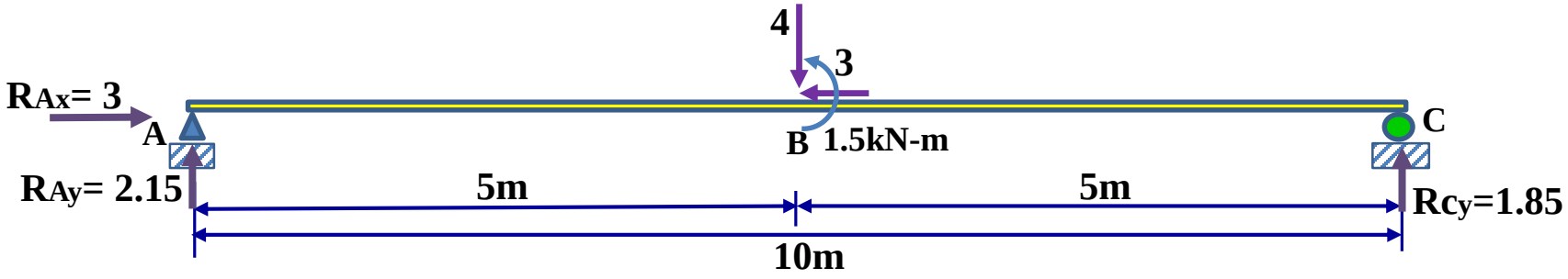
$M_z = 18.5 - 1.85x$



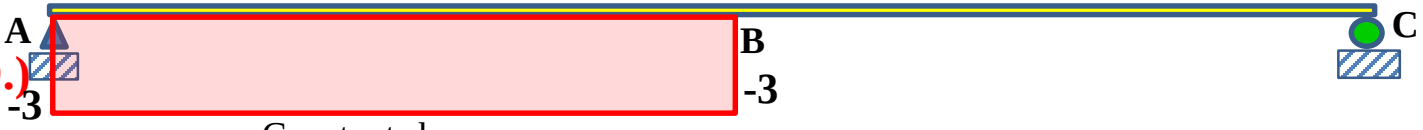
Section 2-2

Load	x = 5	5 < x < 10	x = 10
N=	0	0	0
V=	- 1.85	- 1.85	- 1.85
M=	+ 9.25	+ 18.5 – 1.85x	0

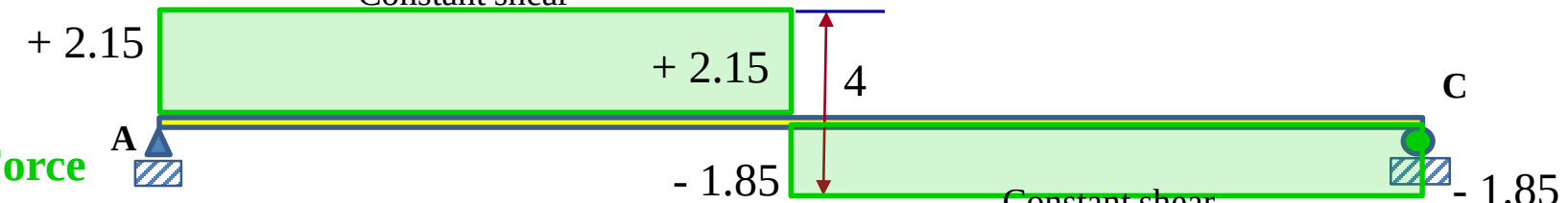
step 3: Draw axial force (N), Shear force (V) and Moment (M) Diagram along beam AB



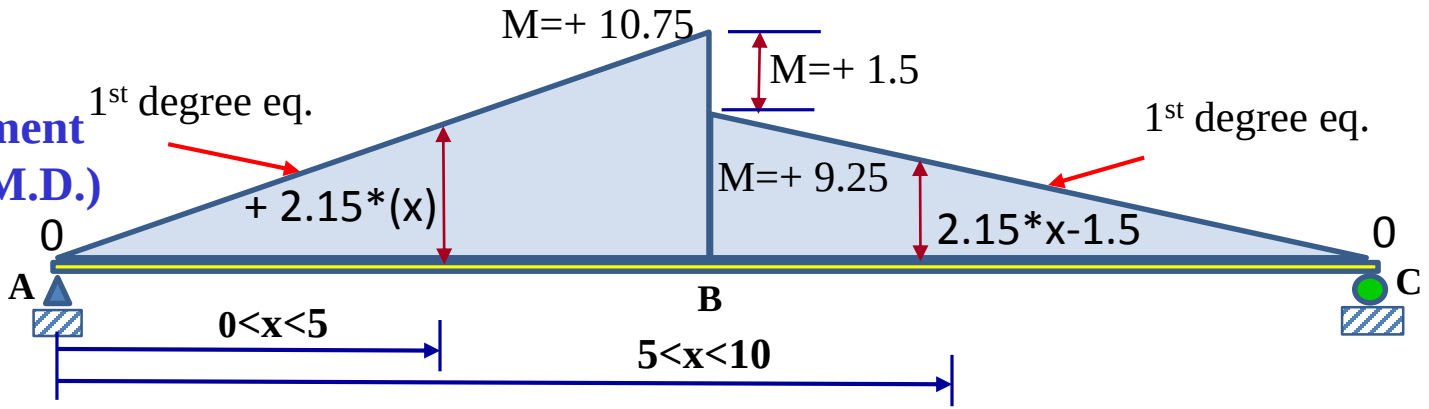
Axial Force Diagram(A.F.D.)



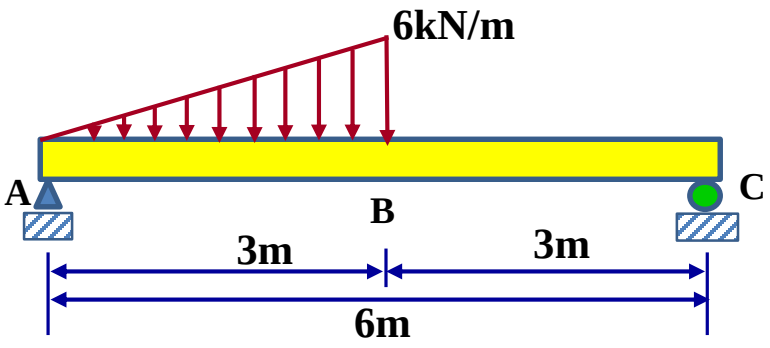
Shear Force Diagram(S.F.D.)



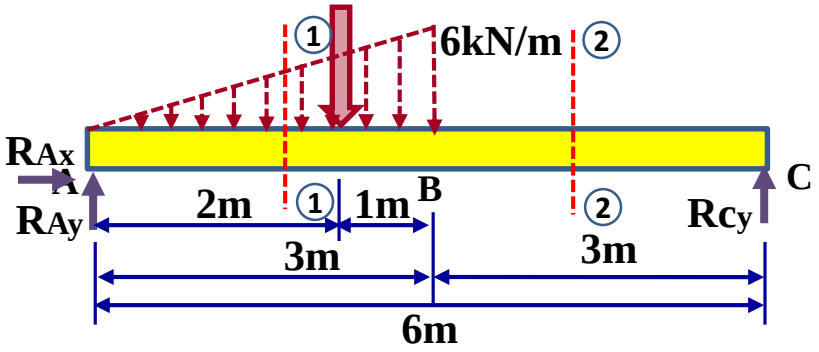
Bending Moment Diagram (B.M.D.)



Example 2-9: Draw axial force, shear force and bending moment diagram of simply supported beam AB spanning 6m carrying partial triangle load increasing from 0 to 6kN/m at span 3m, shown below:



Equivalent load W=6*3/2=9kN



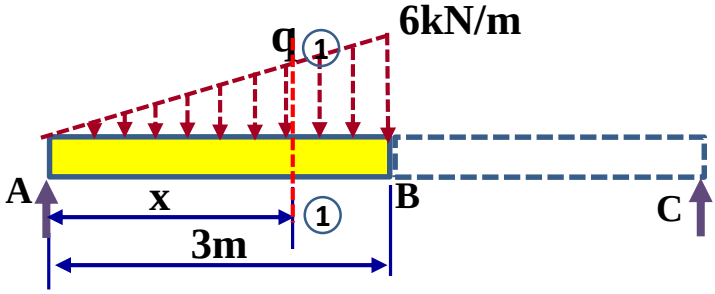
Solution: step 1:

$\Sigma F_x = 0$ ⊕ $R_{Ax} = 0$, $\Sigma M_C = 0$ ⊕, $-R_{Ay} * 6 + 9 * 4 = 0$, $R_{Ay} = 36/6 = 6kN$ ↑

$\Sigma F_y = 0$ ↑⊕, $R_{Ay} + R_{cy} - 9 = 0$ ⟹ $6 + R_{cy} - 9 = 0$, $R_{cy} = 9 - 6 = 3kN$ ↑

For $0 \le x \le 3$ and by similar triangle sides relationship:

$$\frac{q}{x} = \frac{6}{3} \Longrightarrow q = 2*x$$

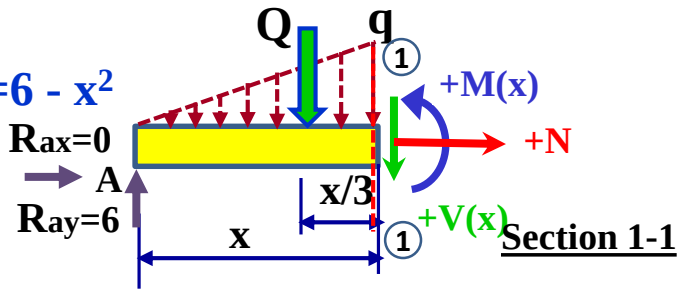


Equivalent load of triangle load span x:

$Q = q * x/2 \Longrightarrow Q = (2x)*x/2 = x^2$

Section 1-1 Part AB ($0 \le x \le 3$):

$\Sigma F_y = 0$, ⊕↑ $- V(x) - Q + 6 = 0$, $V(x) + x^2 - 6 = 0$, **$V(x) = 6 - x^2$**



$\Sigma M_z = 0$ ⊕ $M(x) - 6*x + Q*x/3 = 0$,

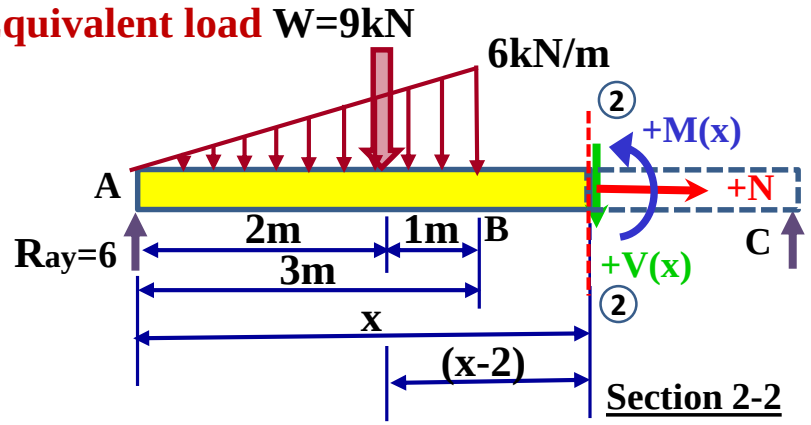
$$M(x) = 6*x - \frac{x^2*x}{3}, \quad \mathbf{M(x) = 6x - \frac{x^3}{3}}$$

For section 2-2 : Part BC (3 ≤ x ≤ 6)

$\Sigma Fy=0 \uparrow + \quad -V(x) - 9 + 6 = 0 \qquad V(x) = -3$

$\Sigma Mz=0 \circlearrowleft + \quad M(x) - 6*x + 9*(x-2) = 0$

$M(x)= 6x -9x + 18, \quad M(x)= 18-3x$



step 2: Define elements

Part AB (0 ≤ x ≤ 3)

Load	x = 0	0 < x < 3	x = 3
N(kN)	0	0	0
V(kN)	+ 6	+ 6 - x^2	- 3
M(kN-m)	0	+ 6x - $x^3/3$	+ 9

Part BC (3 ≤ x ≤ 6)

Load	x = 3	3 < x < 6	x = 6
N(kN)	0	0	0
V(kN)	- 3	- 3	- 3
M(kN-m)	+9	+ 18 - 3x	0

At segment AB $0 \le x \le 3$ and to find M(max)=?, substitute V(x) = 0

$\therefore V(x) = 6 - x^2 = 0, \Longrightarrow \quad x = \sqrt{6} = 2.449m$

Substitute x= 2.449m for $M(x) = 6x - x^3/3$

$\therefore M(max) = 6 * 2.449 - (2.449)^3/3 = 9.7979kN\text{-}m$

step 3: Draw axial force (N), Shear force (V) and Moment (M) Diagram along beam AB

