ASHUR UNIVERSITY

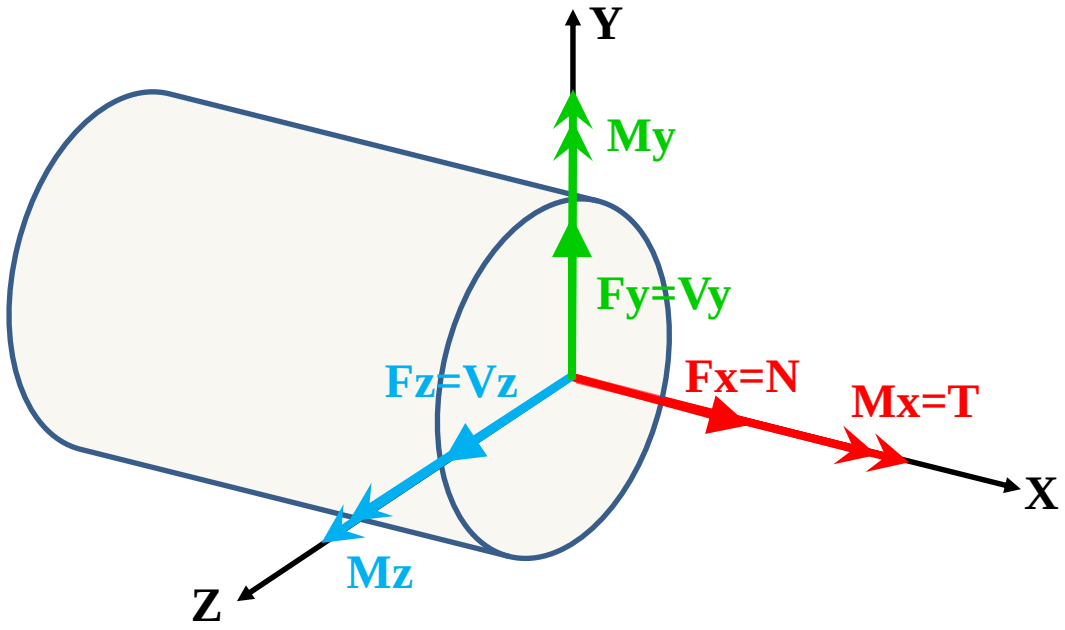
Chapter-2

Part - 1

Axial Force, Shear Force and Bending moment

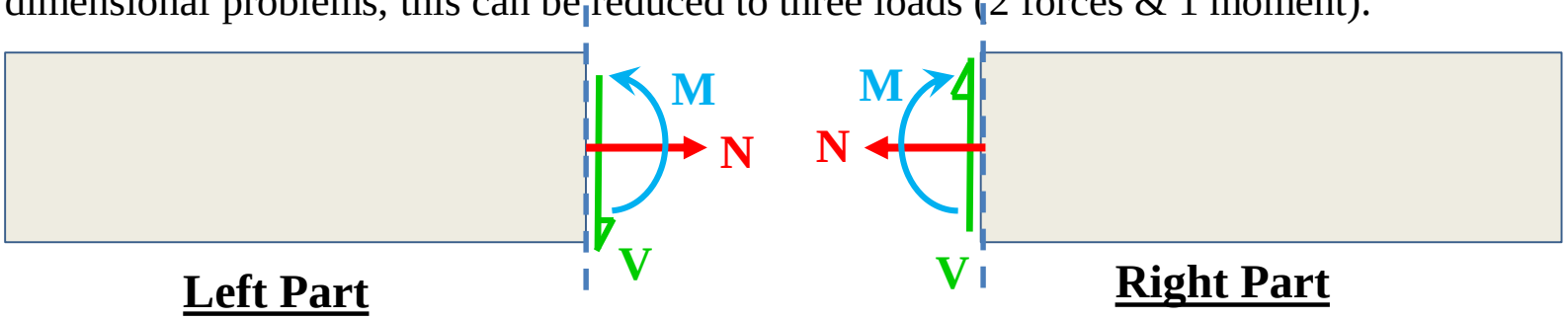
2-1 **Review :**

In general, for a three dimensional problem, there are six loads on any cross section (3 Forces & 3 Moments)



N = Normal Force
V = Shear Force
T = Torque
M = Bending Moment

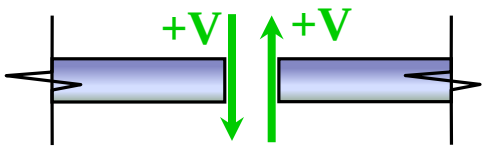
For two dimensional problems, this can be reduced to three loads (2 forces & 1 moment).



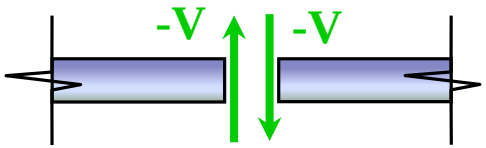
Positive internal load systems (axial force, Shear force and Moment) at section of structural member

Sign Conventions:

1) **SHEAR** is considered **Positive** at section when it tends to **Rotate** the portion of the beam in the **Clock Wise Direction** about an axis through @ point in side the force and normal to the plane of loading, otherwise it is negative.

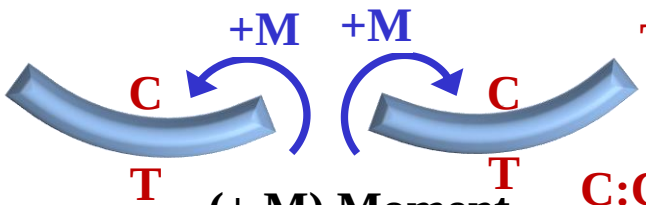


(+ V) Shear



(- V) Shear

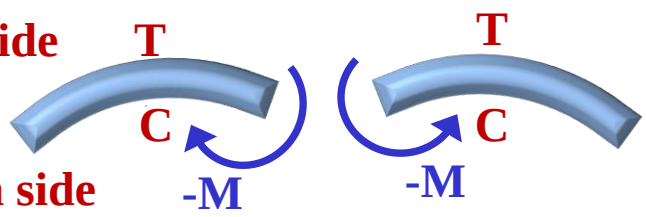
2) **BENDING MOMENT** is considered **Positive** at section when it tends to **bend** the member **Concave @ + ve upward**; otherwise it is negative.



(+ M) Moment

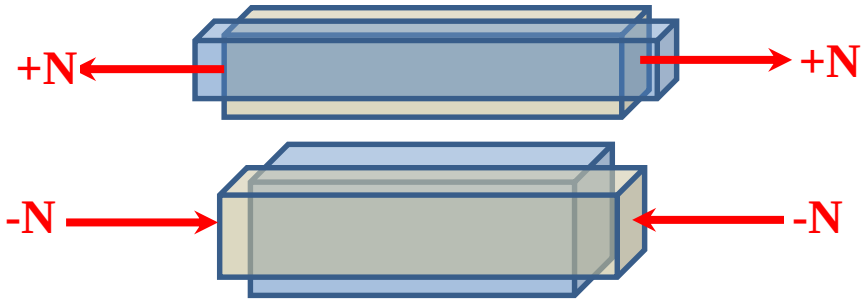
T:Tension side

C:Compresion side



(- M) Moment

3) **AXIAL FORCE**



N=Tension **(+)**, Increase in LENGTH

N = Compression **(-)**, decrease in LENGTH

2-2 Method of Sections :

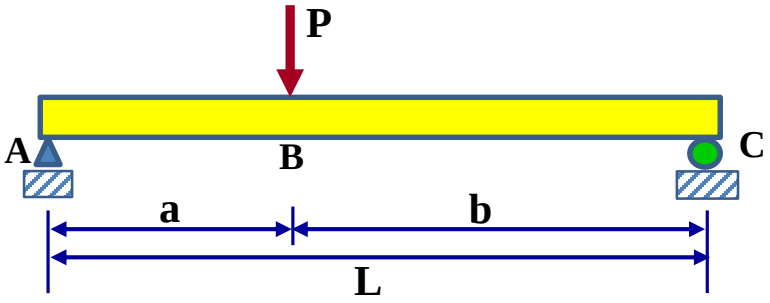
Steps sequence to draw the axial, shear forces and bending moment diagrams by using Method of Sections :

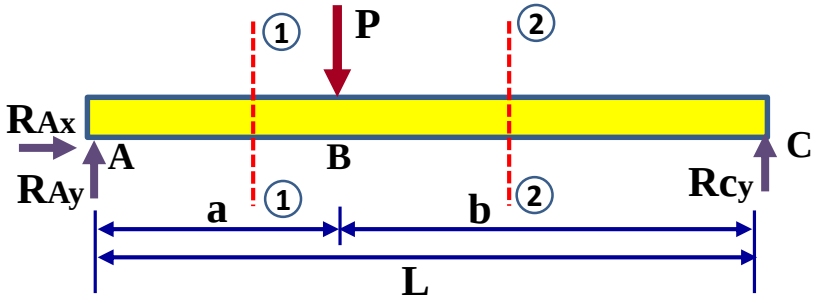
1- Calculate the values and direction of support reactions.

2- Defines the structural element by the locations of sections before and after loadings.

3- Separate each element and apply the equilibrium equations for each element to find axial force, shear force and bending moment with sign conventions.

Example 2-1: Draw axial force, shear force and bending moment diagram of simply supported beam ABC carrying a concentrated load P at any point, shown below:





Solution: step 1:

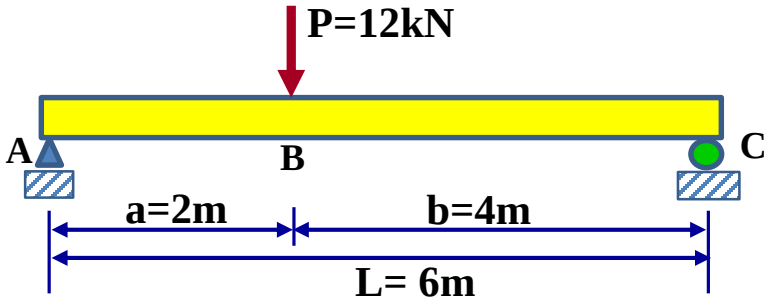
$\Sigma F_x = 0$ (+→) $\quad R_{Ax} = 0$

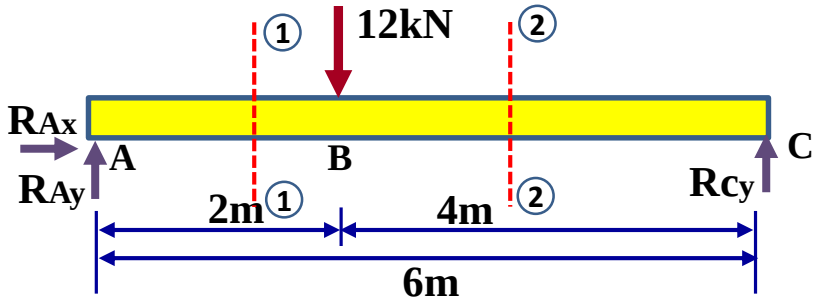
$\Sigma M_c = 0$ (+↻) , $-R_{Ay} * L + P * b = 0$, $\quad R_{Ay} = P*b/L$

$\Sigma \quad F_y = 0$ (+↑) , $R_{Ay} + R_{cy} - P = 0$

$P*b/L + R_{cy} - P = 0$, $\quad R_{cy} = P - P*b/L = P\,(1-b/L) = P/L\,(L-b) = P*a/L$ ↑

Numerical Example: If span **L = 6m, a = 2m, and b = 4m**. Draw axial force, shear force and bending moment diagram of simply supported beam **ABC** carrying a concentrated load **P = 12 kN** at point B, shown below:





Solution: step 1:

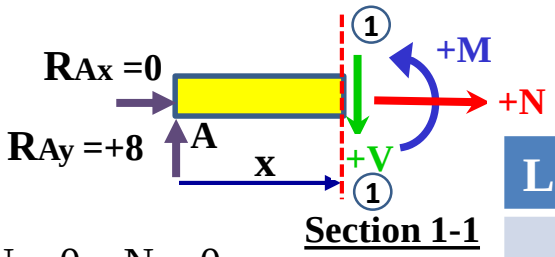
$R_{Ax} = 0$

$\Sigma M_c = 0$ ⟲ $-R_{Ay} * 6 + 12 * 4 = 0$, $R_{Ay} = 48/6 = 8kN$ **or** $R_{Ay} = P*b/L = 12*4/6 = 8kN$ ↑

$\Sigma F_y = 0$ ↑ $8 + R_{cy} - 12 = 0 \rightarrow R_{cy} = 4kN$, **or** $R_{cy} = P*a/L = 12*2/6 = 4kN$ ↑

step 2: Define elements

Part AB ($0 \le x \le a = 2m$)



Sec. 1-1 :

$\Sigma F_x = 0$ → $R_{Ax} + N = 0$, $0 + N = 0$, $N = 0$

$\Sigma F_y = 0$, ↑ $R_{Ay} - V = 0$, $8 - V = 0$, $V = +8$

$\Sigma M_z = 0$, ⟲ $M - R_{Ay}*x = 0$, $M - 8*x = 0$, $M = +8x$

Load	x = 0	0 < x < 2	x =2
N	0	0	0
V	+ 8	+ 8	+8
M	0	+ 8*x	+8*2=16

Part BC (a = 2m ≤ x ≤ L = 6m)

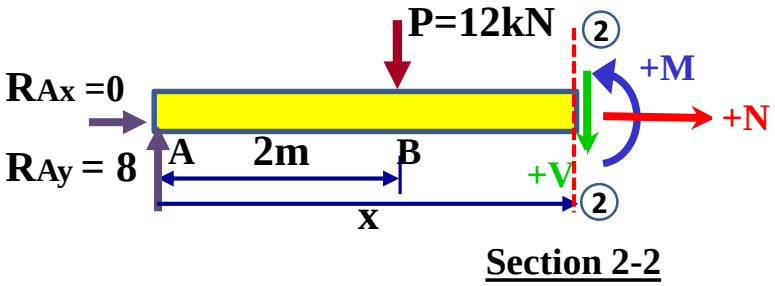
Sec. 2-2 :

ΣFx = 0, → RAx + N =0, 0 + N =0, N = 0

Σ Fy = 0, ↑ 8 -12– V= 0, V = - 4

Σ Mz = 0, ↻ M - 8*x + 12 (x - 2) = 0,

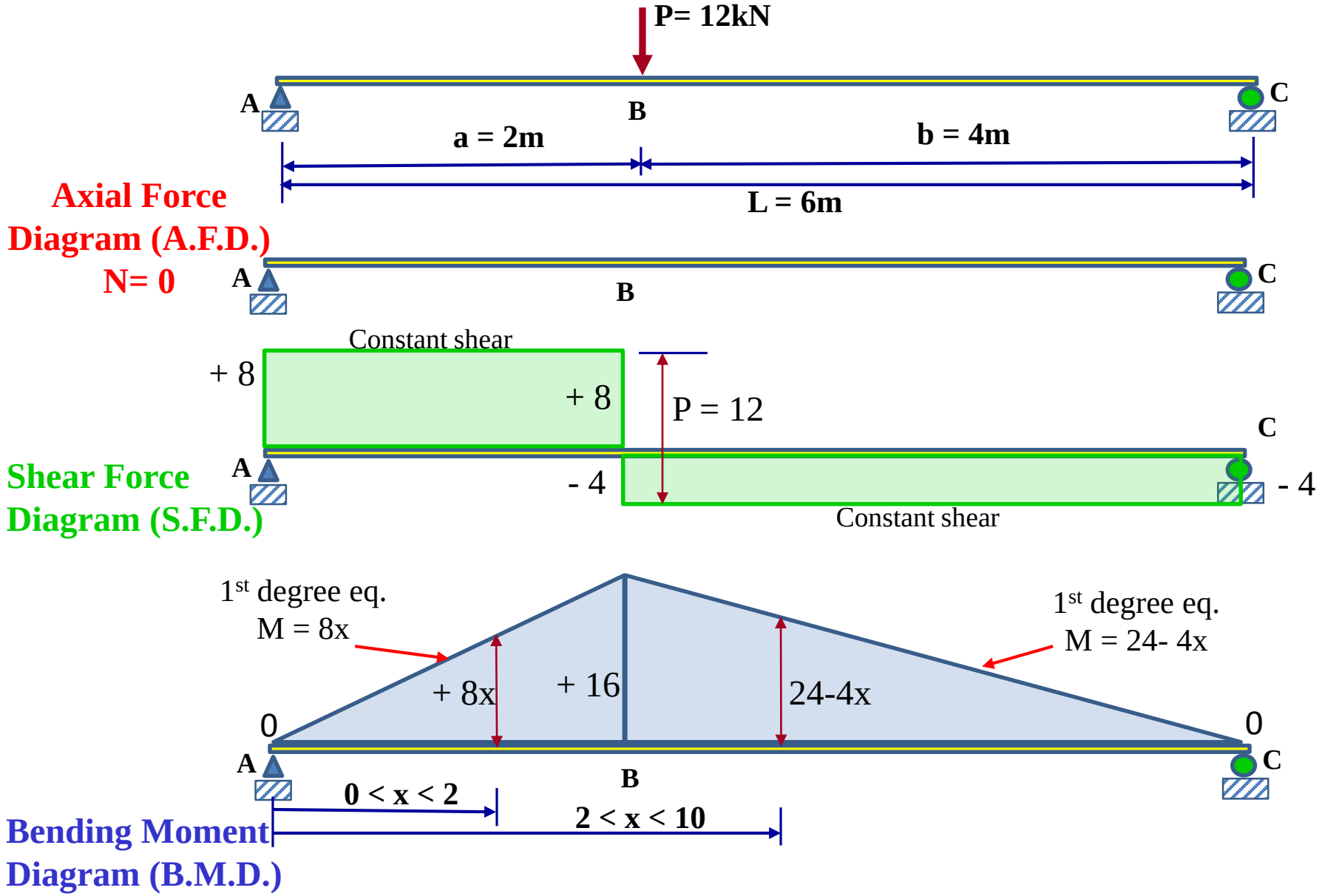
M = 8*x - 12 (x - 2) = 8x -12x + 24 = 24 – 4x



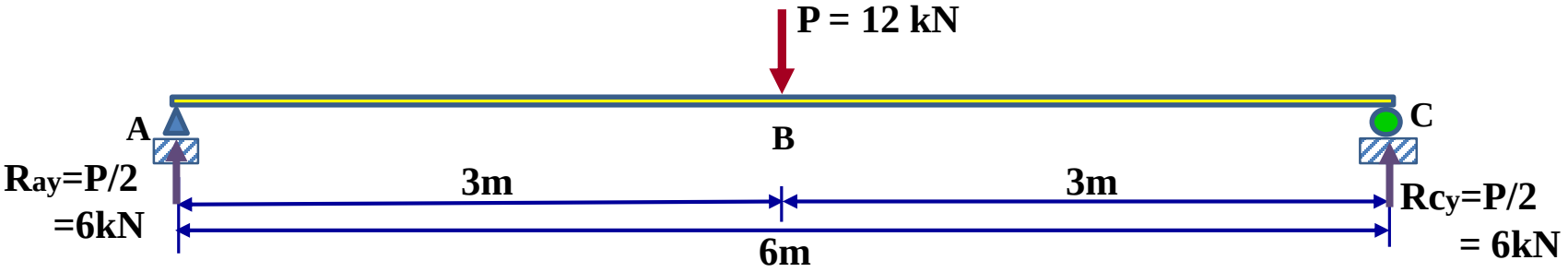
Section 2-2

Load	x = 2	2 < x < 6	x = 6
N	0	0	0
V	- 4	- 4	- 4
M	24-4(2)= 16	24 – 4x	0

step 3: Draw axial force (N), Shear force (V) and Moment (M) Diagram along beam ABC



Remark : If P at mid-span, then a = b = L/2 = 6m

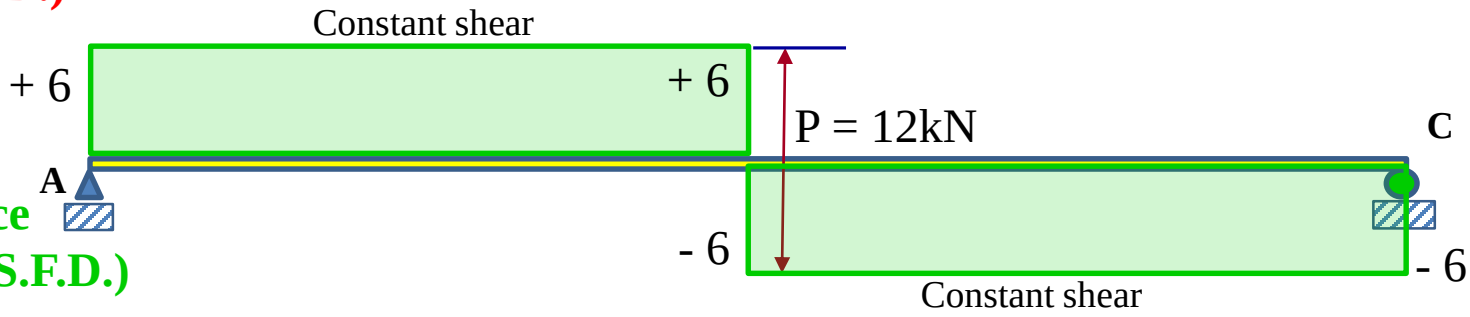


Axial Force Diagram(A.F.D.)

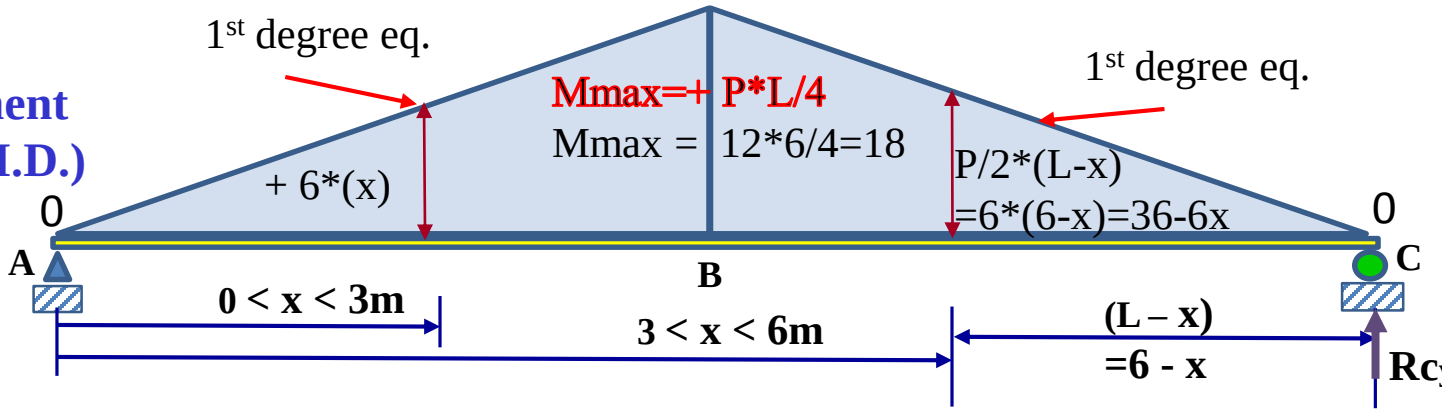
N= 0



Shear Force Diagram (S.F.D.)

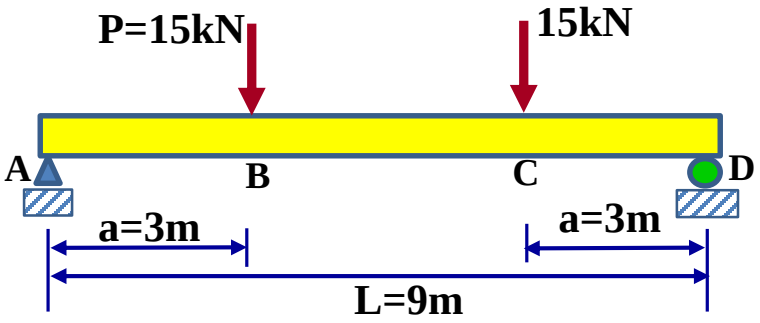


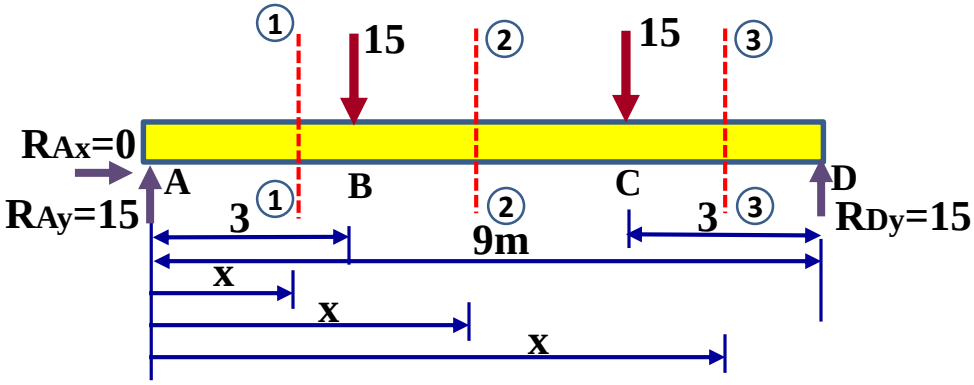
Bending Moment Diagram (B.M.D.)



Remark: This type of simply supported beam loaded with central load used for flexural (bending) test

Example 2-2: Draw axial force, shear force and bending moment diagram of simply supported beam **ABCD** carrying two equal concentrated loads P = 15kN symmetrically placed, shown below:





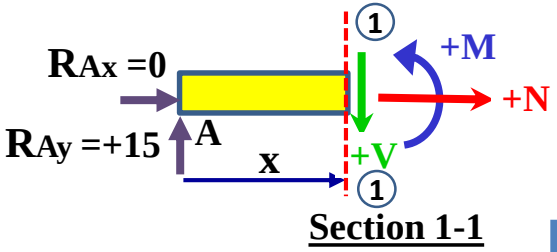
Solution: **step 1:**

Σ Fx = 0, R_{Ax} = 0 and $\Sigma M_A = 0$, R_{Dy}*(9)-15*(3)-15*(6) = 0, 9R_{Dy} – 45 - 90 = 0, R_{Dy} = 135/9 = 15kN

ΣFy = 0, R_{Ay} = 15kN

step 2: Define elements

Part AB (0 ≤ x ≤ 3m)



Section 1-1

Sec. 1-1 :

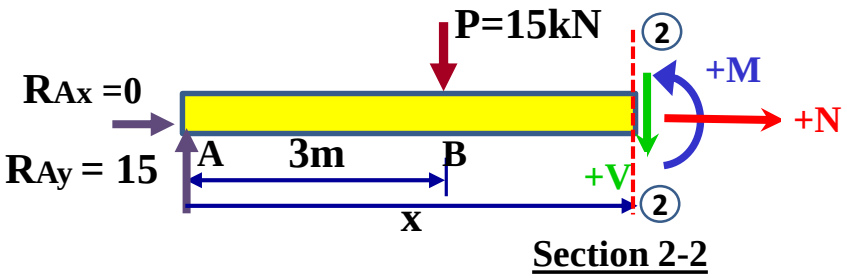
Σ Fx = 0 → R_{Ax} + N= 0, 0 + N = 0, N = 0

Σ Fy = 0, ↑ R_{Ay} – V= 0, 15 – V= 0, V = +15

Σ Mz = 0 , ↻ M - R_{Ay}*x= 0, M – 15*x=0, M = +15x

Load	x = 0	0 < x < 3	x = 3
N	0	0	0
V	+ 15	+ 15	+ 15
M	0	+ 15*x	+ 45

Part BC (3≤ x ≤ 6m)



Sec. 2-2 :

ΣFx = 0, → RAx + N =0, 0 + N =0, N = 0

Σ Fy = 0, ↑ 15 -15– V= 0, V = 0

Σ Mz = 0, ↻ M - 15*x + 15 (x - 3) = 0,

M = 15*x - 15 (x - 3) = 15x -15x + 45 = 45

Load	x = 3	3 < x < 6	x = 6
N	0	0	0
V	0	0	0
M	+ 45	+ 45	+ 45

Part CD (6≤ x ≤ 9m)

Sec. 3-3 :

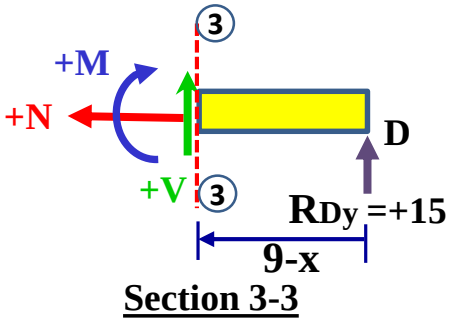
ΣFx = 0, → N = 0

Σ Fy = 0, ↑ 15 + V= 0, V = -15

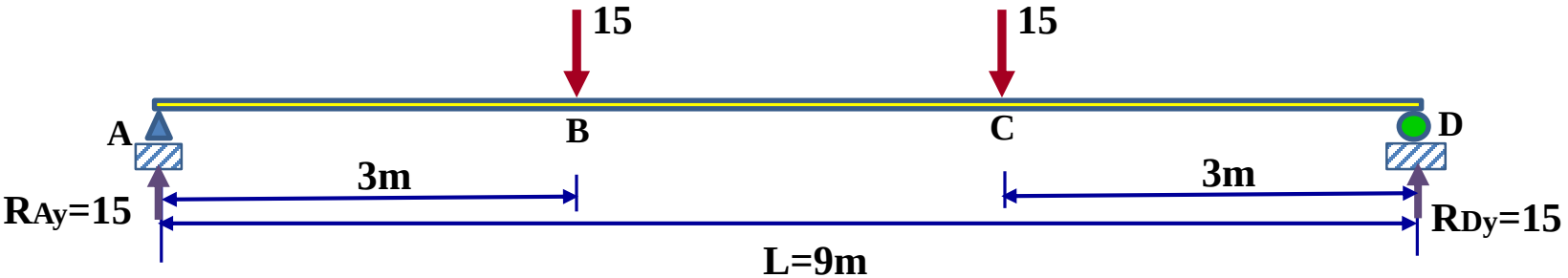
Σ Mz = 0, ↺ M - 15 (9 - x) = 0,

M = 15 (9 - x) = 135 -15x

Load	x = 6	6 < x < 9	x = 9
N	0	0	0
V	- 15	- 15	- 15
M	+ 45	+15*(9-x) =135-15x	0

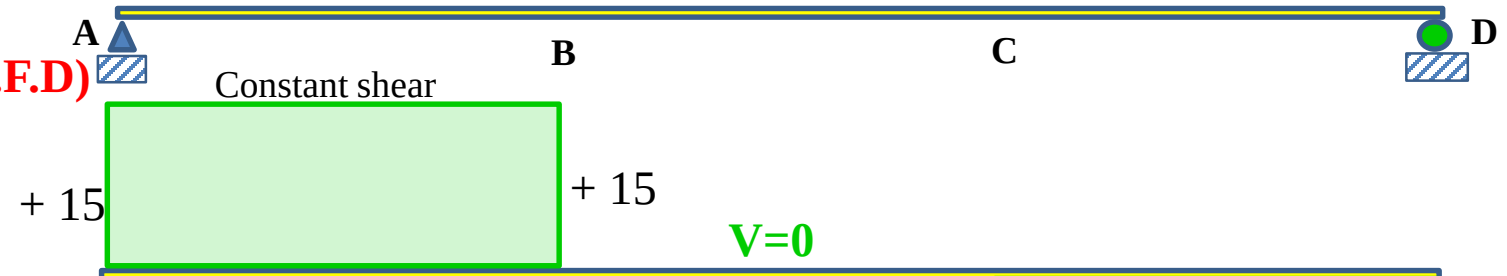


step 3: Draw axial force (N), Shear force (V) and Moment (M) Diagram along beam ABCD



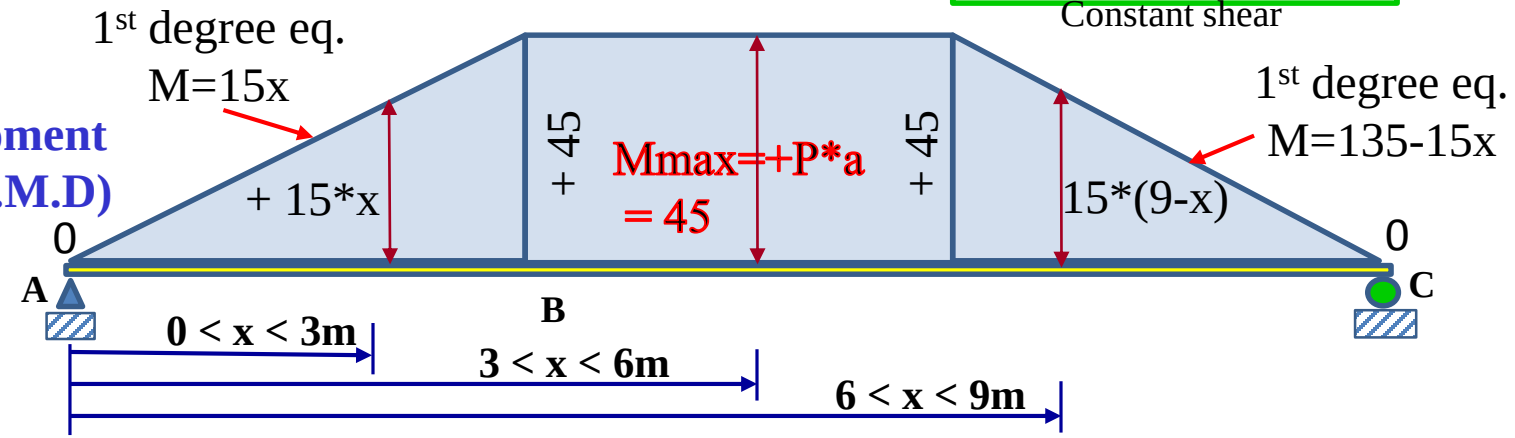
Axial Force Diagram(A.F.D)

N= 0



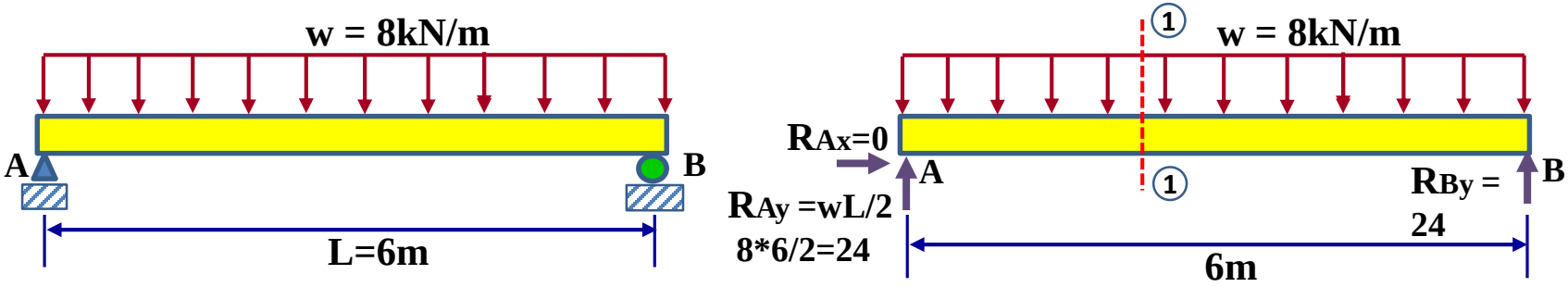
Shear Force Diagram (S.F.D.)

Bending Moment Diagram (B.M.D)



Remark: **This type of simply supported beam loaded with two equal concentrated load used for shearing test**

Example 2-3: Draw axial force , shear force and bending moment diagram of simply supported beam **AB ,** where L = 6m**,** carrying a uniformly full distributed load w = 8kN/m, shown below:



Solution: step 1:

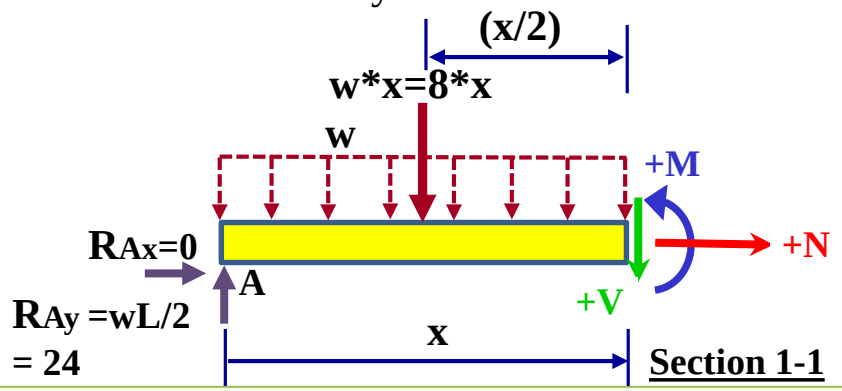
$\Sigma F_x = 0 \Rightarrow R_{Ax} = 0$ and $\Sigma M_A = 0$, $R_{By} * 6 - 8 * 6 * (6/2) = 0, \Rightarrow R_{By} = w * L/2 = 8*6/2 = 24kN$

$\Sigma F_y = 0, \Rightarrow R_{Ay} + 24 - 8*6 = 0 \Rightarrow R_{Ay} = 48 - 24 = 24kN \Rightarrow$ or $R_{Ay} = w*L/2 = 8*6/2 = 24kN$

step 2: Define elements

Part AB ($0 \le x \le 6m$)

Load	x = 0	0 < x < 6	x = 6
N	0	0	0
V	+ 24	+ 24 - 8x	- 24
M	0	+ 24x – 8x(x/2)= 24x – 4x²	0



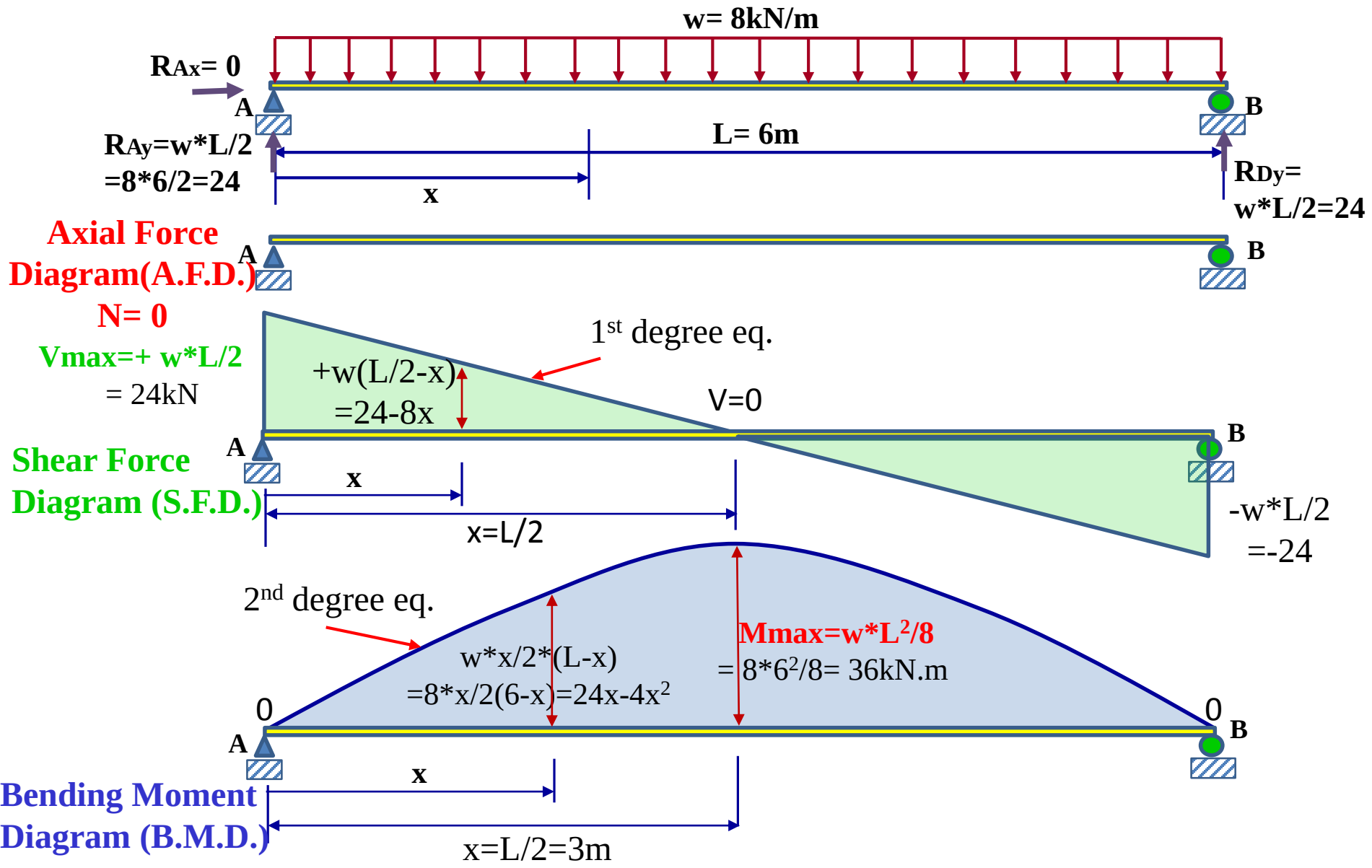
At x = L/2 = 3m (mid-span) :

V = 24 – 8x = 24 – 8*3 = 0 ∴ at x =L/2= 3m, **M = Mmax**

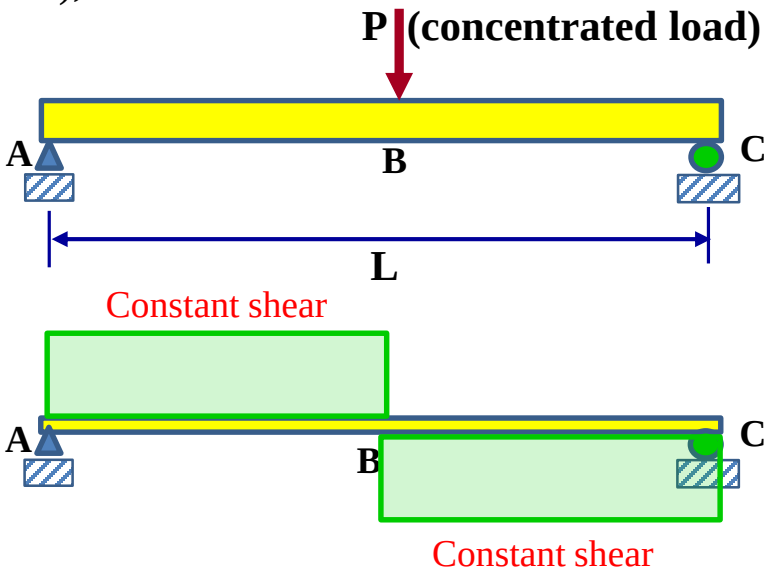
Mmax = wL/2(L/2) – w*L/2(L/4)

= $wL^2/4 - wL^2/8 = wL^2/8$ ∴ **$M_{max} = wL^2/8$**

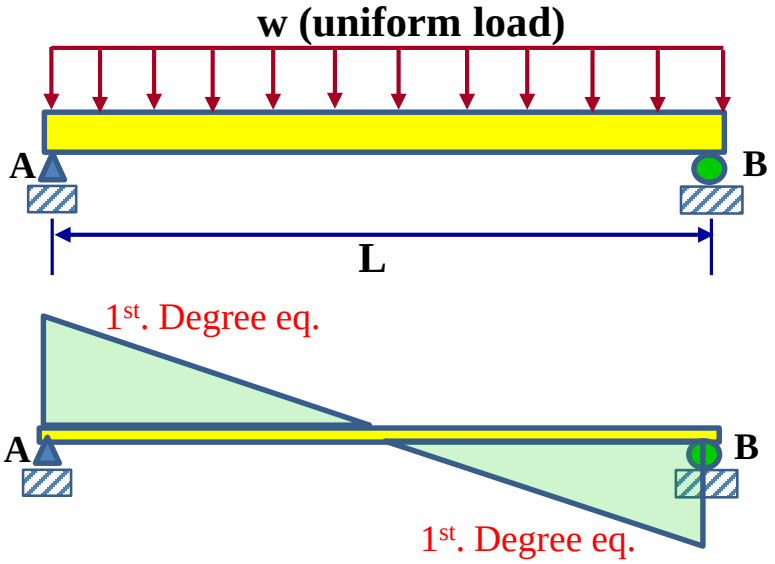
step 3: Draw axial force (N), Shear force (V) and Moment (M) Diagram along beam AB



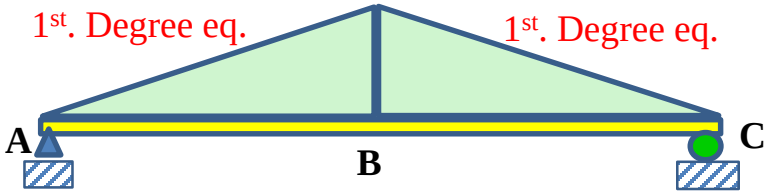
Conclusion: If the applied load is concentrated load, the shear force is constant and bending moment diagram is 1st. Degree equation (linear behavior). If the applied load is uniform load, the shear force is 1st. Degree equation (linear behavior) and bending moment diagram is 2st. Degree equation (parabolic behavior), shown below:



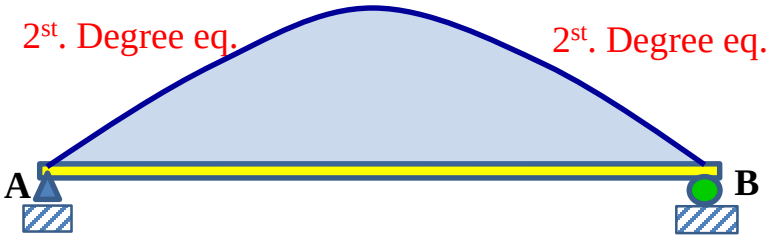
Shear Force Diagram (S.F.D.)



Shear Force Diagram (S.F.D.)



Bending Moment Diagram (B.M.D.)



Bending Moment Diagram (B.M.D.)