Stresses in the bars of Uniformly Tapering Rectangular Sections

The taper is gradual in a shape that represents rectangular faces, and it is of three types:

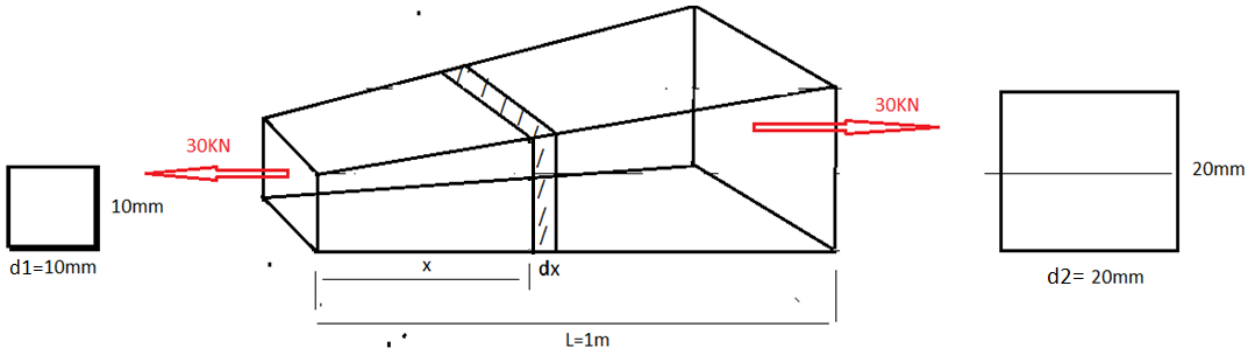
A- When the sides of a rectangle are equals (square).

B- When the sides of the rectangle are of different lengths and similar widths.

C- When the sides of the rectangle are of different lengths and widths

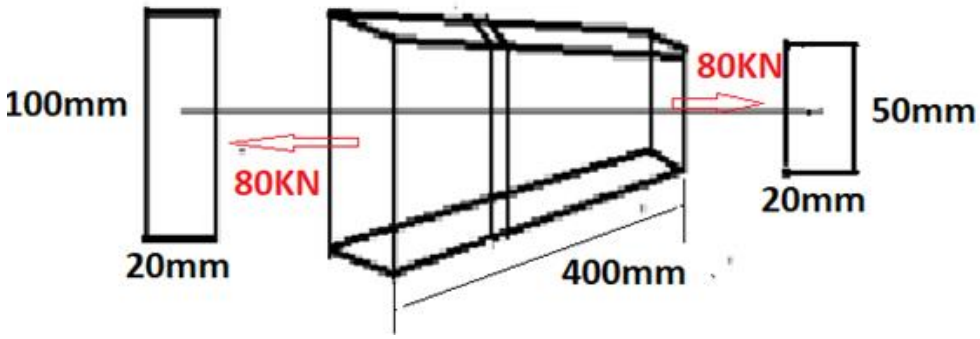
Example 1

An alloy bar of 1m length has square section throughout, with tapers from one end of 10mmx10mm to other end of 20mmx20mm. Find the change of its length due to an axial tensile load of 30KN. Take E for the alloy as 120 GPa.



Example **2**

A steel plate of 20mm thickness tapers uniformly from 100mm to 50 mm in a length of **400mm.** What is the elongation of the plate, if an axial force of 80 Kn acts on it? Take E=200GPa.



$d_X = d_1 - (d_1-d_2)\, X/L = 100 - (100-50)\, X/L = 100 - (100 - 50\, X / 400$

$= 100 - 0.125\, X$

$A = 20x\,(100 - 0.125X)$

$Б_X = P_X / A_X = 80 \times 10^3 / 20 \times (100 - 0.125\, X) = 4 \times 10^3 / (100 - 0.125X)$

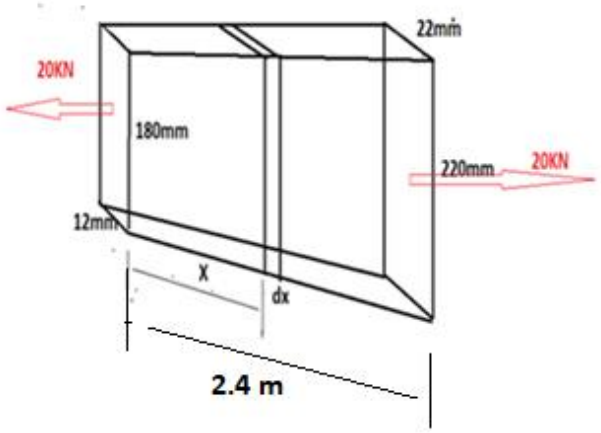
$\varepsilon_X = б_X / E = 4 \times 10^3 / 200 \times 10^3\,(100 - 0.125\, X) = 1 / 50\,(100 - 0.125\, X)$

$\varepsilon = \delta L / L = \delta L / dX$

$$\delta L = \varepsilon \times dX = \int_0^{400} dX \,/\, 50(100-0.125X) = 1/50 \int_0^{400} d\,X / (\,100 - 0.125X)$$

Example 3

A steel plate 22mm thick and 220mm wide at one end and tapers uniformaly to 12mm thick and 150mm wide at other end. Determine the elongation under a pull of 20 KN when the length of the plate is 2.4m, take E 205 GPa.



Width of a small strip $d_X = 180 - (180 - 220)X / 2400 = 180 + X/60$

Thickness of a small strip $d_X = 12 - (12 - 22)X / 2400 = 12 + X/240$

$Area_X = Width_X \times Thickness_X = (180 + X/60)\times(12 + X/240)$

$$\delta L = P_X L_X / A_x E$$

$$= 20\times10^3 \, xdX / (180+X/60)(12+X/240)\times 205\times10^3$$

$$\delta L = \int_0^{2400} 20\, dx / (180+X/60)(12+X/240)\times 205$$