Mechanics of Materials, Stress and Strain part 2

Presented by professor Ali Hussein Ataiwi

Exampl1

A load of 5KN is to be raised with help of a steel wire. Find the minimum diameter of the steel wire if the stress is not to exceed 100MPa.

Sol.

P=5KN=5x$10^{3,}$ б=100MPa=100N/mm^2 , d=diameter of the wire in mm.

Б=P/A=5x10^3 / π/4xd^2

d^2 =6.366x!0^3 /100 d= 7.98 mm

Ex. 2

A hallow cylinder 2 m long has an outside diameter of 50mm and inside diameter of 30 mm. If the cylinder is carrying a load of 25 KN, find the stress in the cylinder, also find the deformation of the cylinder, if the value of modulus of elasticity for the cylinder material is 100GPa.

Sol.

L= 2m=2x$10^{3,}$ P=25 KN=25x10^3 N, E= 100GPa=100x10^3 MPa=100x10^3 N/mm^2 .ππ

A=π/4x (D^2 – d^2) = [50^2 – 30^2]=1257mm^2

Stress in cylinder б = P/ a =25x10^3 / 1257= 19.9 MPa

Deformation of cylinder δL=P.L /A. E =(25x$10^{3)\cdot}$ (2x10^3) / 1257 x (100x 10^3) =0.4mm

Deformation of a Body Due to Self Weight

Consider a bar AB hanging freely under its own weight as shown in Fig. 2.4.

Let l = Length of the bar.

A = Cross-sectional area of the bar.

E = Young's modulus for the bar material.

and w = Specific weight of the bar material (density)

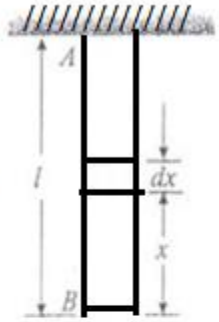


Fig.2.4

Now consider a small section dx of the bar at a distance x from B. We know that weight of the bar for a length of x,

$$P = wAx$$

$\therefore$ Elongation of the small section of the bar, due to weight of the bar for a small section of length x,

$$= \frac{Pl}{AE} = \frac{(wAx).dx}{AE} = \frac{wx.dx}{E}$$

Total elongation of the bar may be found out by integrating the above equation between zero and l. Therefore total elongation,

$$\delta l = \int_0^l \frac{wx.dx}{E}$$

$$= \frac{w}{E}\int_0^l x.dx$$

$$= \frac{w}{E}\left[\frac{x^2}{2}\right]_0^l$$

or $$\delta l = \frac{wl^2}{2E} = \frac{Wl}{2AE} \quad \ldots(\because W = wAl = \text{Total weight})$$

Note: From above result we find that the deformation of the bar, due to its own weight, is equal to half of the deformation if the same body is subjected to a direct load equal to the weight of the body

Ex.3

A copper alloy wire of 1.5mm diameter and 30 m long is hanging freely from a tower. What will be its elongation due to self-weight? Take specific weight of copper and its modulus of elasticity as 89.2 KN/m^2 and 90 GPa respectively.

Sol.

$\delta L = wL^2 / 2E = (89.2 \times 10^{-6}) \times (30 \times 10^{-6}) / 2 \times (90 \times 10^{3}) = 0.45$ mm

Ex.4

An alloy of wire 2mm^2 cross sectional area and 12N weight hangs freely under its own weight. Find the maximum length of the wire, if its extension is not to exceed 0.6 mm. Take E for the wire material as 150 GPa.

Ans.

$\delta L = WL / 2AE$

$0.6 = 12xL / 2x2x (150x10^3) = 0.02x10^{-3}\ L$

$L = 0.6 / 0.02x10^{-3} = 30000$ mm= 30m

Principle of superposition

Sometimes a body is exposed to a number of forces that affect and cause a deformation on its ends as well as on the sections along its length. In this case, the forces are divided and their effect is taken on the sections that make up the body and according to the points of impact. The resulting deformation of the body is equal to the algebraic sum of the deformations of the divided sections. This principle is called the principle of superposition .The equation for this principle can be written as follows:

$\delta L = P.L / A . E$

$= 1 / A . E (P_1 . L_1 + P_2 . L_2 + P_3 . L_3 + \ldots\ldots\ldots + P_n . L_n)$

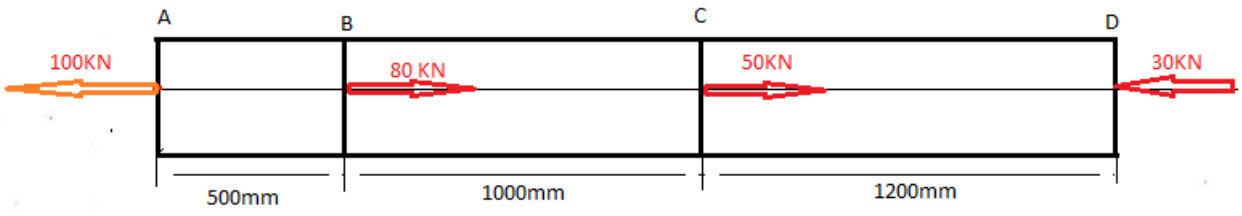
$P_1 =$ Force acting on section 1

$L_1 =$ Length of section 1

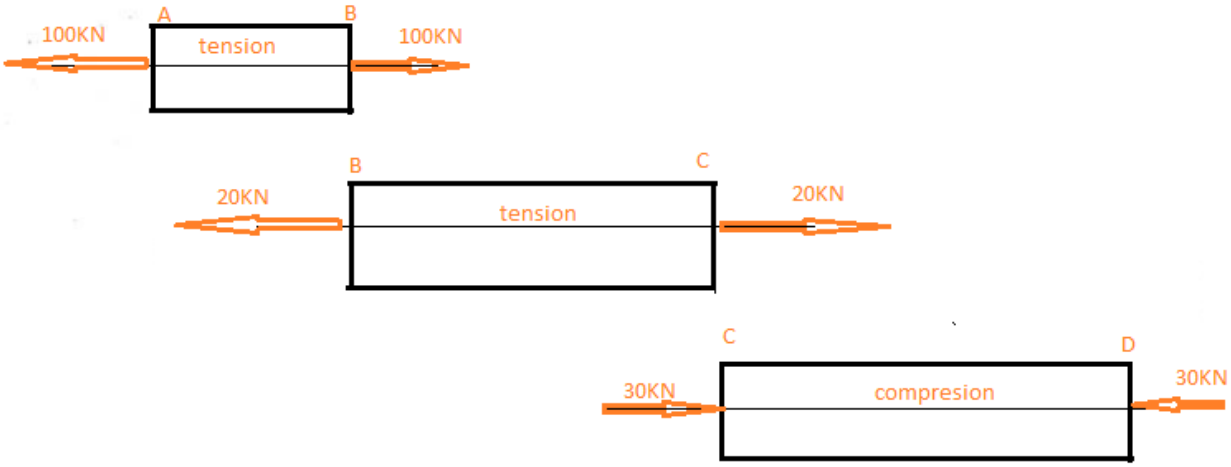
(+) is taken for tension and (-) for compression

Ex. 5

A brass bar having cross- sectional area of 500 mm^2 is subjected to axial forces as shown in Fig. Find the total elongation of the bar. Take E=80 GPa



Sol.

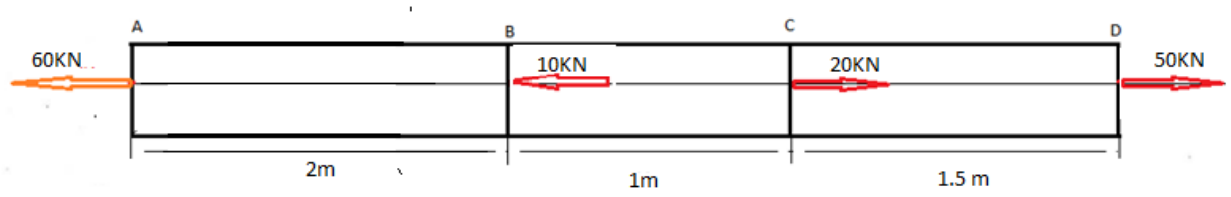


Take Tension as (+) and Compression as (-)

$\delta L = 1/AE\ (P_1 . L_1 + P_2 . L_2 + P_3 . L_3) = 1/50x80\ [(100x500) + (2x1000) - (30x1200)] = 0.85$ mm

Home work

A steel rod ABCD 4.5 m long and 25mm in diameter is subjected to the forces as shown in the following figure



Strains in bars of varying sections

3.1. Introduction

In the last chapter, we have discussed the procedure of obtaining stresses and strains in the bars of uniform cross-sectional area. But sometimes we come across bars of varying sections in which we are required to find out the stresses and strains. The procedure for finding out the stresses and strains in same sections is slightly different.

3.2. Types of Bars of Varying Sections

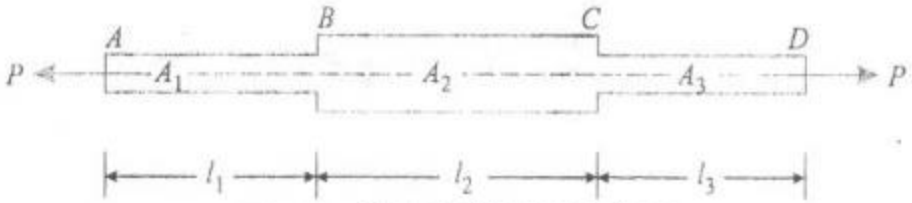
Though there are many types of bars of varying sections, in the field of strength of materials yet the following are important from the subject point of view :

1. Bars of different sections
2. Bars of uniformly tapering sections
3. Bars of composite sections.

Now we shall study the procedure for the stresses and strains in the above mentioned bars in the following pages.

3.3. Stresses in the Bars of Different Sections

Sometimes a bar is made up of different lengths having different cross-sectional areas as shown in Fig. 3.1.



Bars of different sections

In such cases, the stresses, strains and hence changes in lengths for each section is worked out separately as usual. The total changes in length is equal to the sum of the changes of all the individual lengths. It may be noted that each section is subjected to the same external axial pull or push.

Let
P = Force acting on the body,
E = Modulus of elasticity for the body,
l_1 = Length of section 1,
A_1 = Cross-sectional area of section 1,
l_2, A_2 = Corresponding values for section 2 and so on.

We know that the change in length of section 1.

$$\delta l_1 = \frac{Pl_1}{A_1 E} \qquad \text{Similarly} \qquad \delta l_2 = \frac{Pl_2}{A_2 E} \qquad \text{and so on}$$

∴ Total deformation of the bar,

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3 + \ldots\ldots\ldots$$

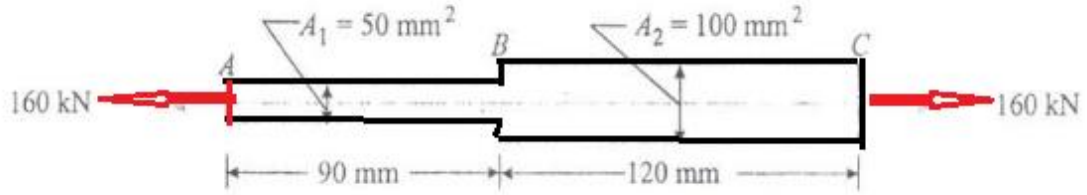
$$= \frac{Pl_1}{A_1 E} + \frac{Pl_2}{A_2 E} + \frac{Pl_3}{A_3 E} + \ldots\ldots\ldots$$

$$= \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} + \ldots\ldots\ldots\right)$$

NOTE. Sometimes, the modulus of elasticity is different for different sections. In such cases, the total deformation,

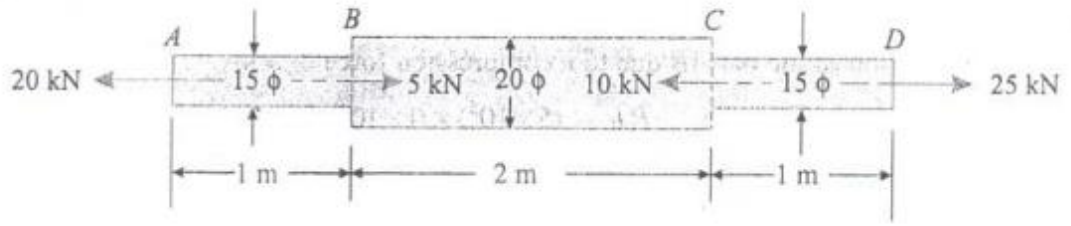
$$\delta l = P\left(\frac{l_1}{A_1 E_1} + \frac{l_2}{A_2 E_2} + \frac{l_3}{A_3 E_3} + \ldots\ldots\ldots\right)$$

EXAMPLE 3.1. *An automobile component shown in Fig. 3.2 is subjected to a tensile load of 160 kN.*



Determine the total elongation of the component, if its modules of elasticity is 200 GPa.

EXAMPLE 3.7. *A steel bar ABCD 4 m long is subjected to forces as shown in Fig. 3.8.*



Find the elongation of the bar. Take E for the steel as 200 GPa.

SOLUTION. Given : Total length of steel bar (L) = 4 m = 4×10^3 mm ; Length of first part (l_1) = = 1×10^3 mm ; Diameter of first part (d_1) = 15 mm ; Length of second part (l_2) = 2 m = 2×10^3 ; Diameter of second part (d_2) = 20 mm ; Length of third part (l_3) = 1 m = 1×10^3 mm ; Diameter hird part (d_3) = 15 mm and modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm^2.

We know that area of the first and third parts of the bar,

$$A_1 = A_3 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (15)^2 = 177 \text{ mm}^2$$

area of the middle part of the bar

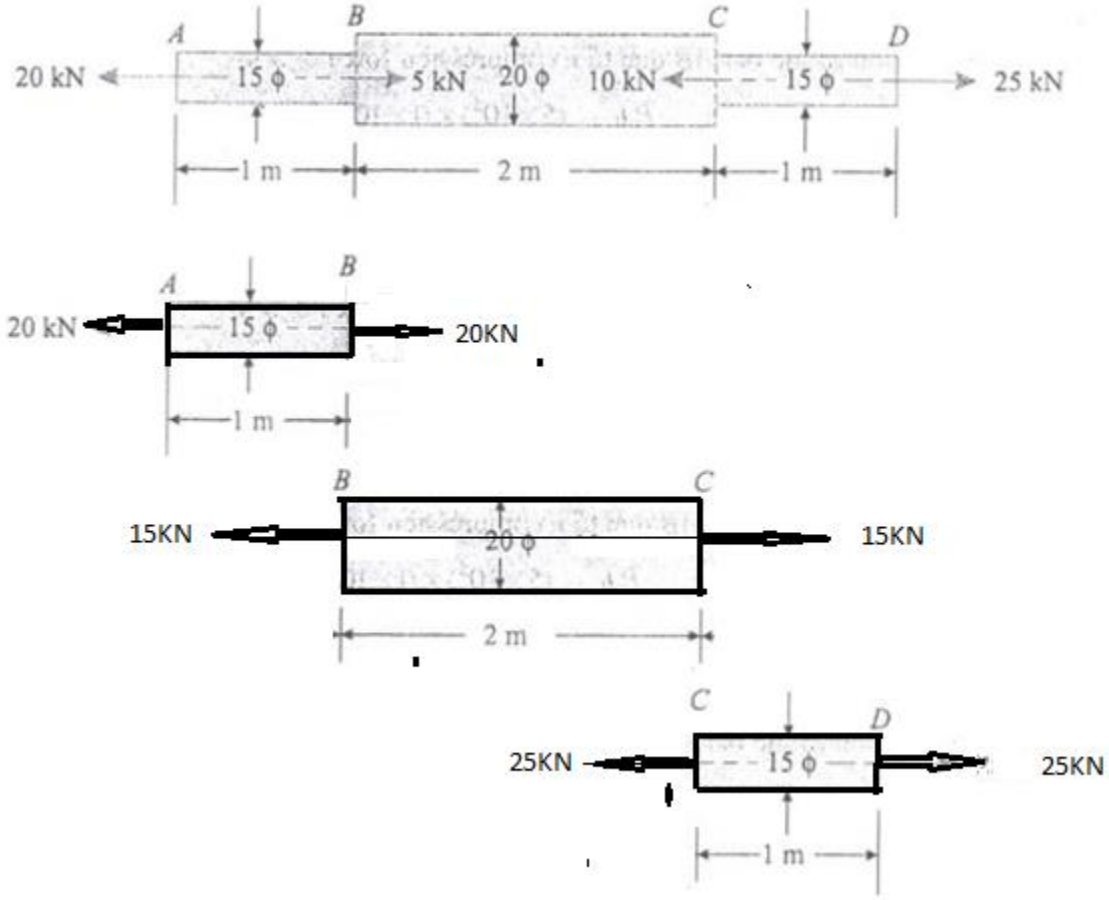
$$A_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (20)^2 = 314 \text{ mm}^2$$

For the sake of simplification, the force of 25 kN acting at D may be split up into two forces of 15 and 10 kN respectively. Similarly the force of 20 kN acting at A may also be split up into two ces of 15 kN and 5 kN respectively.

Now it will be seen that the bar $ABCD$ is subjected to a tensile force of 15 kN, part BC is jected to a compressive force of 5 kN and the part CD is subjected to a tensile force of 10 kN as wn in Fig. 3.9.

We know that elongation of the bar $ABCD$ due to a tensile force of 15 kN,

$$\delta l_1 = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]$$



$\delta L = \delta L_{AB} + \delta L_{BC} + \delta L_{CD/4/}$

$\delta L = 20x1x10^2 / 200x\pi/4 \times 15^2 + 15 \times 2 \times 10^3 / 200x \pi/4 \times 20^2 + 25x1x10^3 /200x\pi/4x\ 15^2$

$\delta L = 1.73$ mm