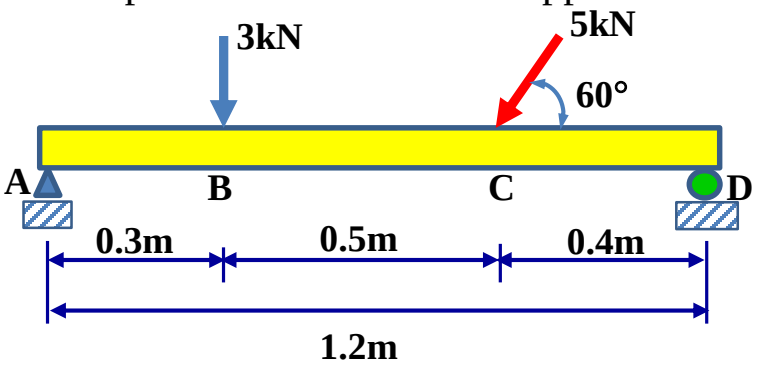
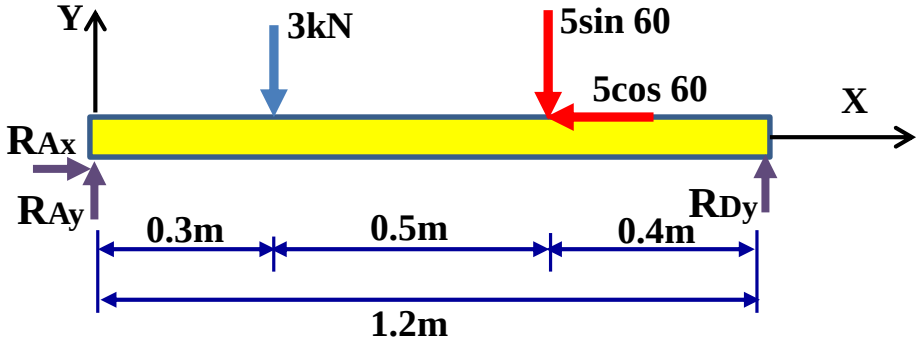
1-14 Calculation of Support Reactions:

The conditions of statically equilibrium, are used to calculate reactions at supports in structures so long as the support system is **statically determinate**, (there are a number of unknowns equal to the number of equations of statically equilibrium).

Example 1-1: Calculate the support reactions in the simply supported beam **ABCD** shown below:





Structure Free-Body Diagram

Solution:

$\Sigma F_x = 0 \Rightarrow$ $R_{Ax} - 5*\cos 60 = 0$, $R_{Ax} = 2.5$ kN →

$\Sigma M_D = 0$ ↺

$-R_{Ay} * (1.2) + 3 * (0.9) + 5 \sin 60 * (0.4) = 0$, $R_{Ay} = 3.7$ kN ↑

$\Sigma F_y = 0$ ⇑ , $R_{Ay} + R_{Dy} - 3 - 5 \sin 60 = 0$

$3.7 + R_{Dy} - 3 - 5 \sin 60 = 0$

$R_{Dy} = 3.6$ kN ↑

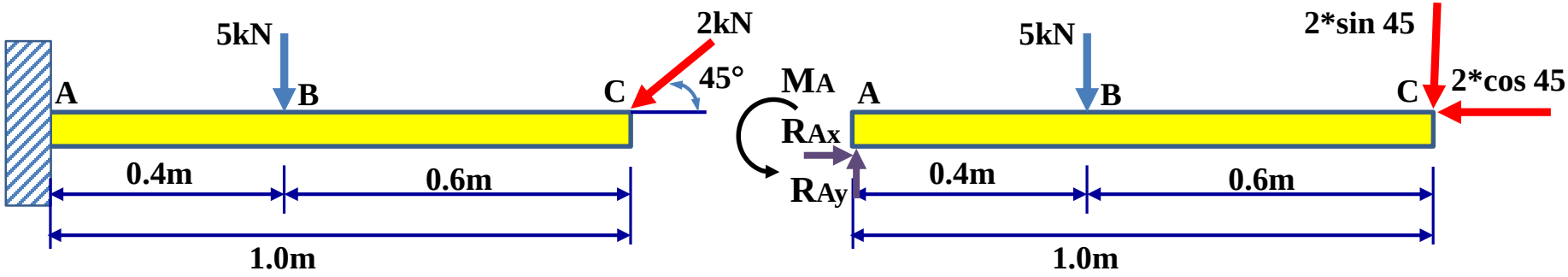
ASHUR UNIVERSITY

Chapter – 1

Part - 3

Introduction

Example 1-2: Calculate the support reactions in the cantilever beam **ABC** shown below:



Structure Free-Body Diagram

Solution:

$\Sigma F_x = 0$ ⟹ $R_{Ax} - 2*\cos 45 = 0$, $R_{Ax} = 1.4$ kN →

$\Sigma F_y = 0$ ⇑ , $R_{Ay} - 5 - 2 \sin 45 = 0$, $R_{Ay} = 6.4$ kN ↑

$\Sigma M_A = 0$ ⟲ , $M_A - 5 * (0.4) - 2 \sin 45 * (1.0) = 0$, $M_A = 3.4$ kN-m

the resultant reaction at a support is:

$R_A\text{^}2 = R_{Ax}\text{^}2 + R_{Ay}\text{^}2$

$R_A\text{^}2 = 1.4\text{^}2 + 6.4\text{^}2$, $R_A = 4.5$ kN

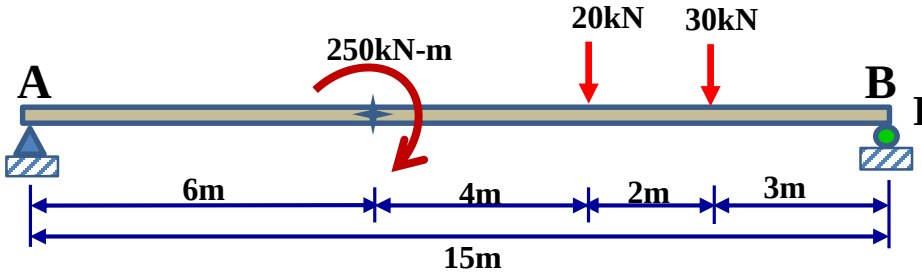
The inclination of **R,** to

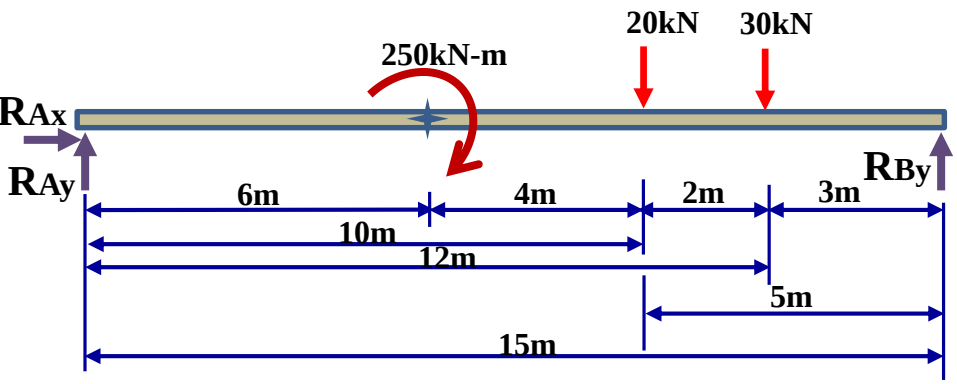
$\tan \theta = R_{Ax} / R_{Ay}$

$= 1.4 / 6.4 = 0.676$

From which $\theta = 34.0°$

Example 1-3: Find reactions of simply supported beam shown below:





Structure Free-Body Diagram

Solution:

$\Sigma F_X = 0 \Rightarrow R_{Ax} = 0$

$\Sigma M_A = 0$

$R_{By} * (15) - 30 * (12) - 20 * (10) - 250 = 0$

$R_{By} = 54$ kN ↑ Positive sign means assumed direction of reaction was corrected

$\Sigma M_B = 0$

$-R_{Ay} * (15) - 250 + 20 * (5) + 30 * (3) = 0$

$R_{Ay} = -4$ kN Negative sign means assumed direction of reaction was not corrected

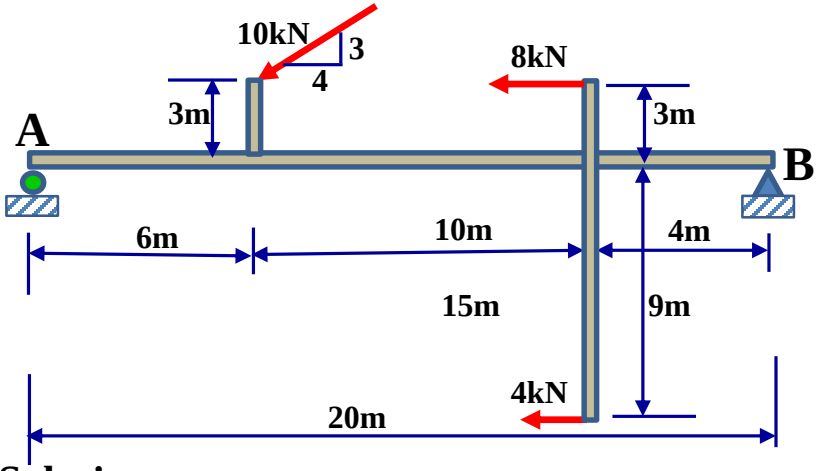
$\therefore R_{Ay} = 4$ kN ↓

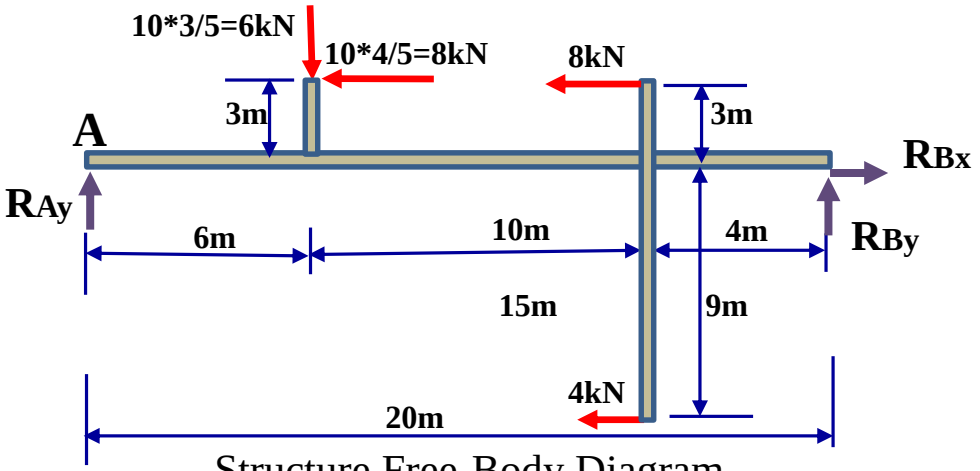
Check: $\Sigma F_y = 0$ ↑+

$54 - 20 - 30 - 4 = 0$

$54 - 54 = 0 \quad \therefore$ OK

Example 1-4: Calculate reactions at supports for the beam shown below:





Structure Free-Body Diagram

Solution:

$\Sigma F_x = 0$ $\Rightarrow$, $-8 - 8 - 4 + R_{Bx} = 0$, $R_{Bx} = 20$ kN

$\Sigma M_A = 0$ ⟲

$+ R_{By} * (20) + 8 * (3) + 8 * (3) - 4 * (9) - 6 * (6) = 0$

$R_{By} = 1.2$ kN ↑ Positive sign means assumed direction of reaction was corrected

$\Sigma M_B = 0$ ⟲

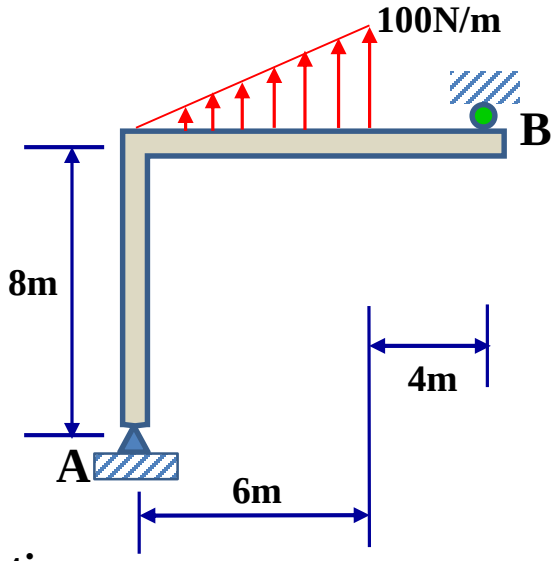
$-R_{Ay} * (20) + 6 * (14) + 8 * (3) + 8 * (3) - 4 * (9) = 0$

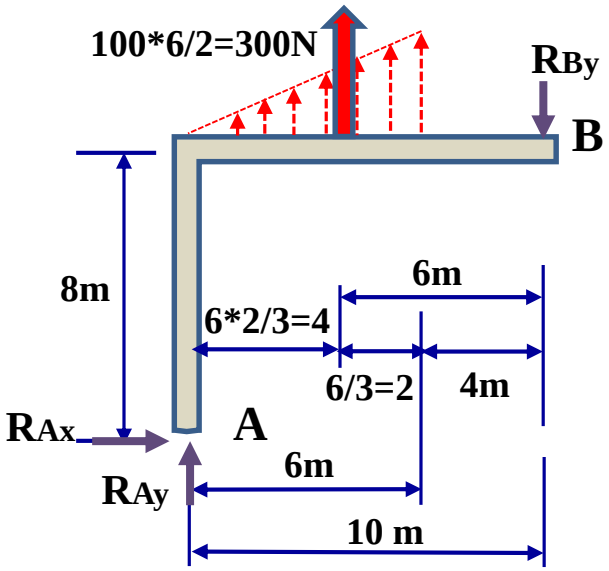
$R_{Ay} = 4.8$ kN ↑

Check: $\Sigma F_y = 0$ ↑ , $4.8 - 6 + 1.6 = 0$, $6 - 6 = 0$, ∴ OK

Check: $\Sigma F_x = 0$ $\Rightarrow$, $20 - 8 - 8 - 4 = 0$, $20 - 20 = 0$ ∴ OK

Example 1-5: Calculate reactions at supports for the frame shown below:





Structure Free-Body Diagram

Solution:

$\Sigma F_x = 0 \Rightarrow R_{Ax} = 0$

$\Sigma M_A = 0$

$-R_{By} * 10 + 300 * 4 = 0$

$R_{By} = 120$ N ↓ Positive sign means assumed direction of reaction was corrected

$\Sigma M_B = 0$

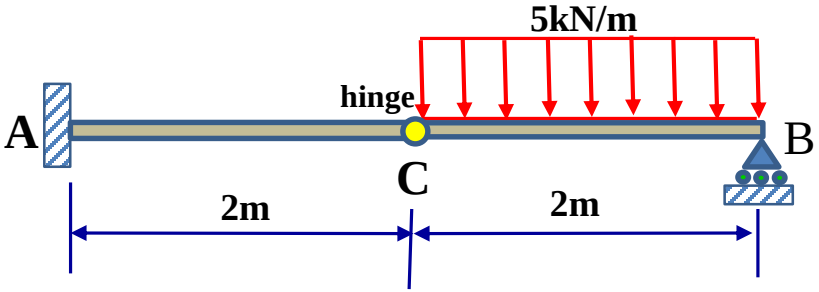
$-R_{Ay} * 10 - 300 * 6 = 0$

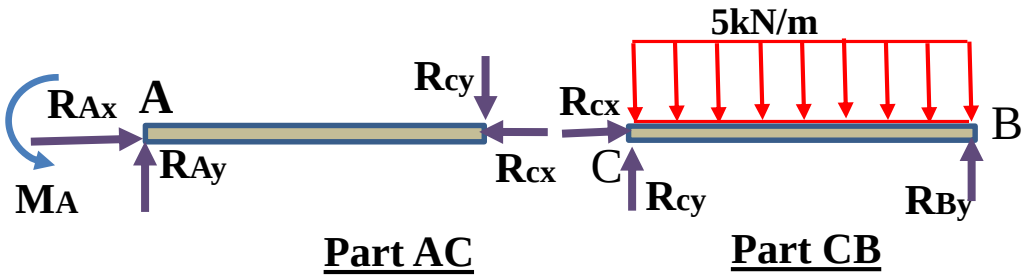
$R_{Ay} = -180$ N Negative sign means assumed direction of reaction was not corrected

$\therefore R_{Ay} = 180$ N ↓

Check: $\Sigma F_y = 0$ ↑+ , $-120 + 300 - 180 = 0$, $-300 + 300 = 0$, $\therefore$ OK

Example 1-6: Calculate reactions at supports for the beam shown below:





Solution: **In this case there are grater than 3 unknown reactions (RAx, RAy, MA and RBy), so existing internal hinge add another equilibrium equation (MC = 0)**

Part CB

$\Sigma F_x = 0$ → $R_{cx} = 0$

$\Sigma M_C = 0$ ↺, $-5*2*(1) + R_{By}*(2) = 0$, $R_{By} = 5$ kN

$\Sigma F_y = 0$ ↑, $R_{cy} + 5 - 5*(2) = 0$, $R_{cy} = 5$ kN

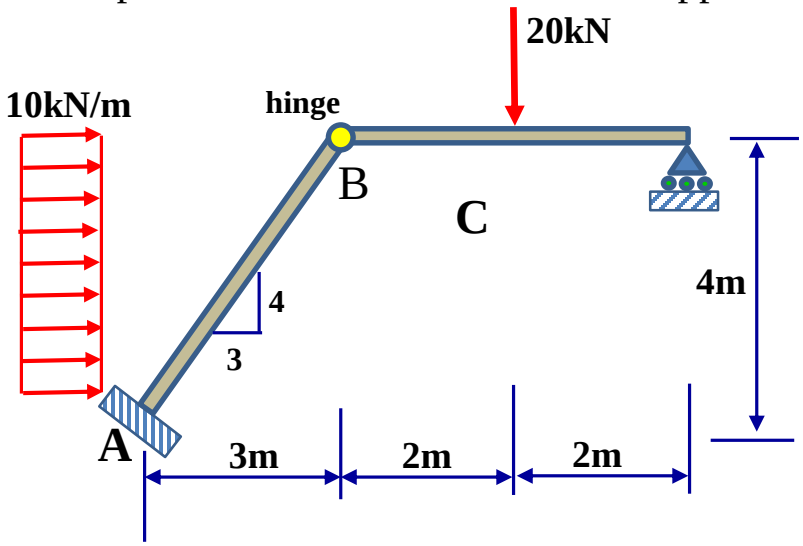
Part AC

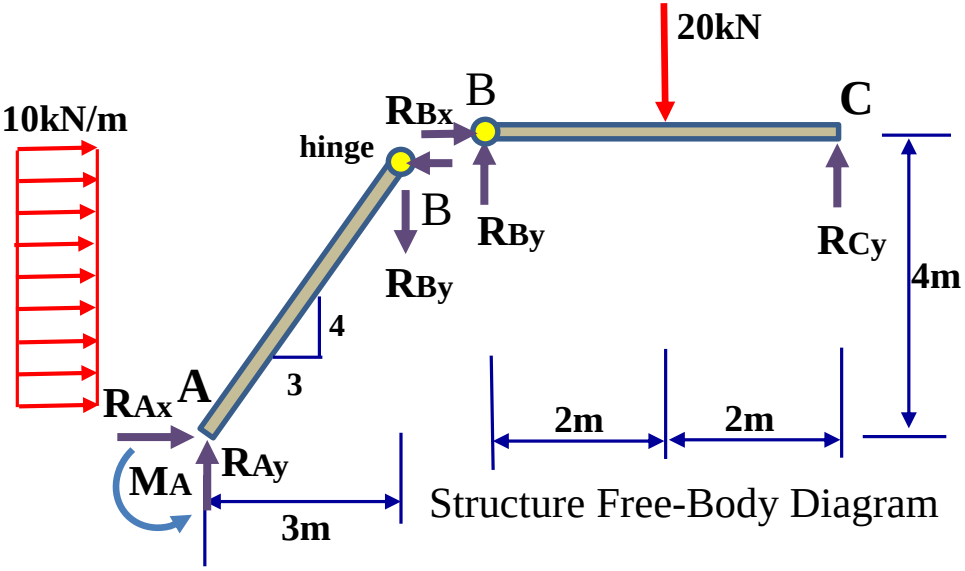
$\Sigma F_x = 0$ →, $R_{Ax} - R_{cx} = 0$, $R_{Ax} - 0 = 0$, $R_{Ax} = 0$

$\Sigma F_y = 0$ ↑, $R_{Ay} - 5 = 0$, $R_{Ay} = 5$ kN

$\Sigma M_A = 0$ ↺, $M_A - 5*(2) = 0$, $M_A = 10$ kN-m

Example 1-7: Calculate reactions at supports for the frame shown below:





Structure Free-Body Diagram

<u>Solution:</u> In this case there are grater than 3 unknown reactions (RAx, RAy, MA and Rcy), so existing internal hinge add another equilibrium equation ($M_B = 0$)

<u>Part BC</u>

$\Sigma F_x = 0$, $R_{Bx} = 0$

$\Sigma M_B = 0$, $-20*(2) + R_{cy}*(4) = 0$, $R_{cy} = 10$ kN

$\Sigma F_y = 0$, $R_{By} - 20 + 10 = 0$, $R_{By} = 10$ kN

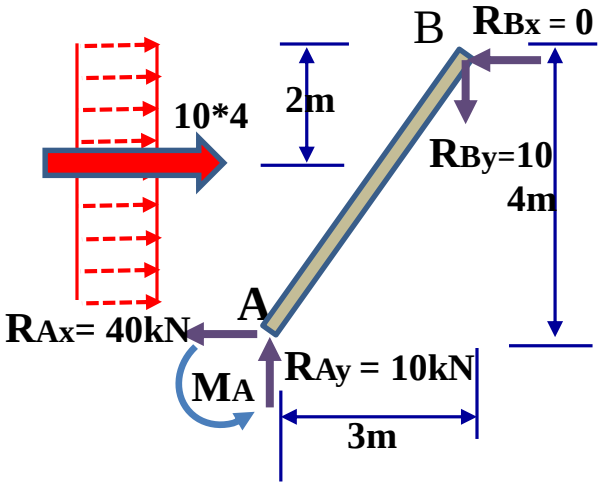
<u>Part AB</u>

$\Sigma F_x = 0$, $10*(4) + R_{Ax} = 0$, $R_{Ax} = -40$ kN

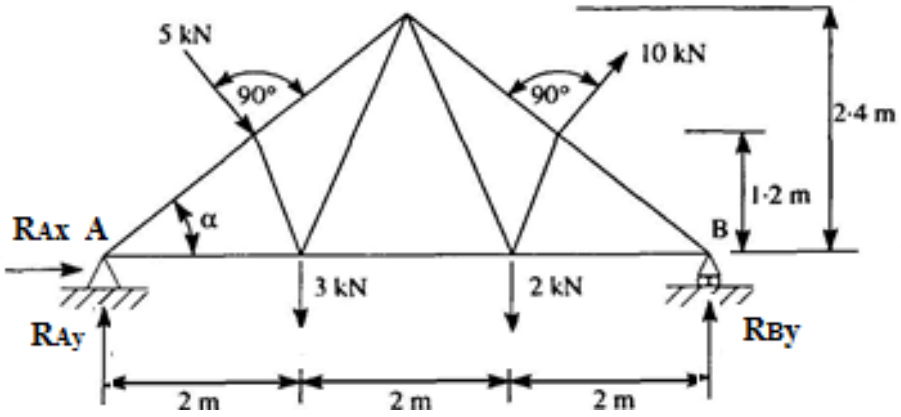
$\Sigma F_y = 0$, $R_{Ay} - 10 = 0$, $R_{ay} = 10$ kN

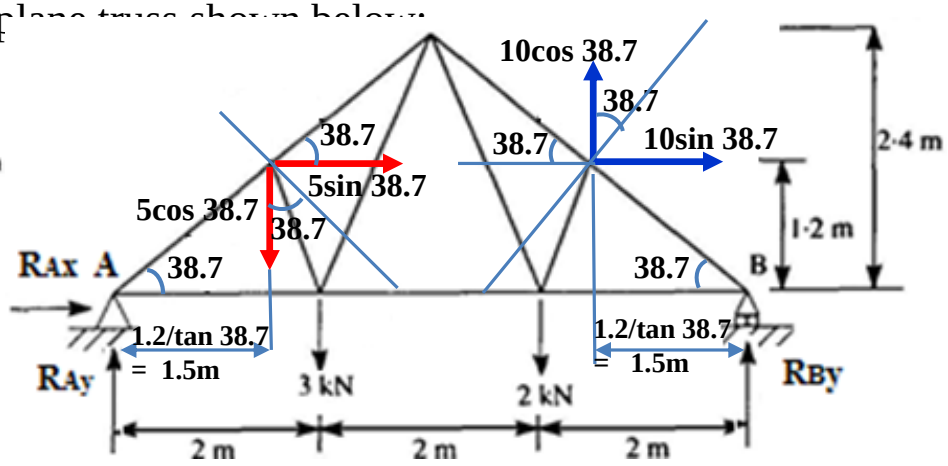
$\Sigma M_B = 0$, $M_A - 10*(3) - 40*4 + 10*4*(2) = 0$

$M_A = +110$ kN-m



Example 1-8: Calculate reactions at supports in the plane truss shown below:





Solution:

$\alpha = \tan^{-1}(2.4/3) = 38.7°$

$\Sigma F_x = 0 \rightarrow R_{Ax} + 5\sin 38.7 + 10\sin 38.7 = 0, \quad R_{Ax} = -9.4$ kN

$\Sigma M_B = 0$ ⟲

$-R_{Ay}(6) + 5\cos 38.7(4.5) - 5\sin 38.7(1.2) - 10\cos 38.7(1.5) - 10\sin 38.7(1.2) + 3(4) + 2(2) = 0$

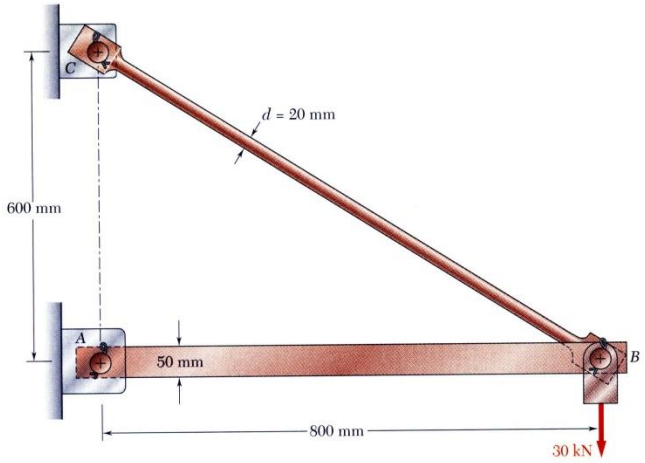
$R_{Ay} = 1.8$ kN

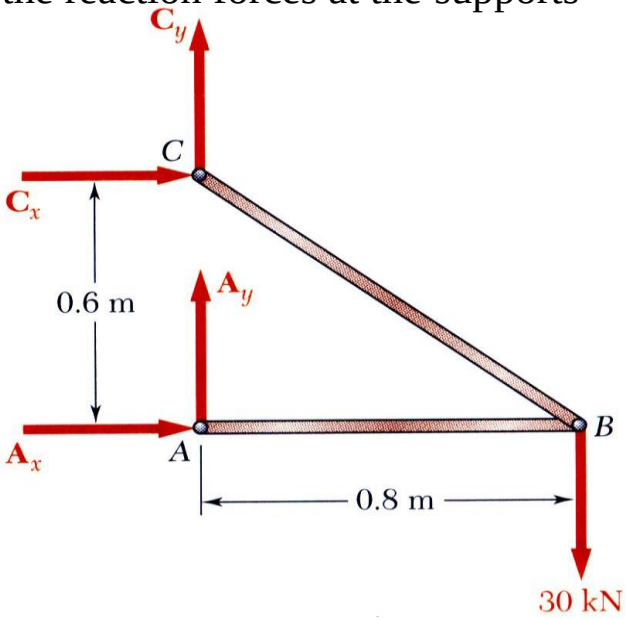
$\Sigma F_y = 0$ ↑ , $R_{Ay} + R_{By} - 5\cos 38.7 + 10\cos 38.7 - 3 - 2 = 0$

$1.8 + R_{By} - 3.90215 + 7.80430 - 5 = 0$

$R_{By} = -0.70215$ kN ↓

Example 1-9: The structure is designed to support a 30 kN load, The structure consists of a boom and rod joined by pins (zero moment connections) at the junctions and supports Perform a static analysis to determine the internal force in each structural member and the reaction forces at the supports





Structure Free-Body Diagram

Solution: Conditions for static equilibrium:

$$\sum M_C = 0 = A_x(0.6\,\text{m}) - (30\,\text{kN})(0.8\,\text{m})$$

$$A_x = 40\,\text{kN}$$

$$\sum F_x = 0 = A_x + C_x$$

$$C_x = -A_x = -40\,\text{kN}$$

$$\sum F_y = 0 = A_y + C_y - 30\,\text{kN} = 0$$

$$A_y + C_y = 30\,\text{kN}$$

- A_y and C_y can not be determined from these equations

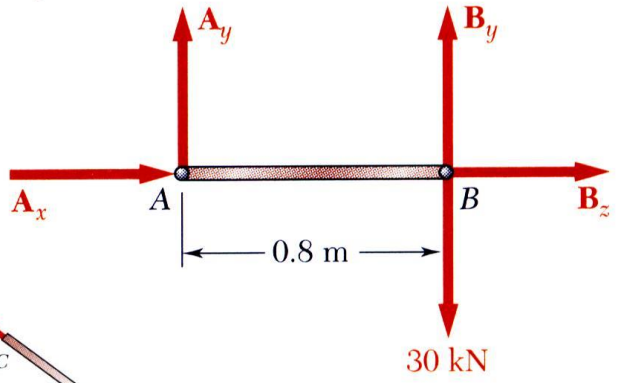
- In addition to the complete structure, each component must satisfy the conditions for static equilibrium
  - Consider a free-body diagram for the boom:

$$\sum M_B = 0 = -A_y(0.8\,\text{m})$$

$$A_y = 0$$

substitute into the structure equilibrium equation

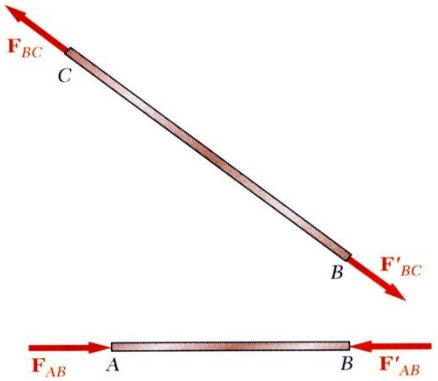
$$C_y = 30\,\text{kN}$$



- Results:

$$Ax = 40\,\text{kN} \rightarrow \quad C_x = 40\,\text{kN} \leftarrow \quad C_y = 30\,\text{kN} \uparrow$$

Reaction forces are directed along boom and rod

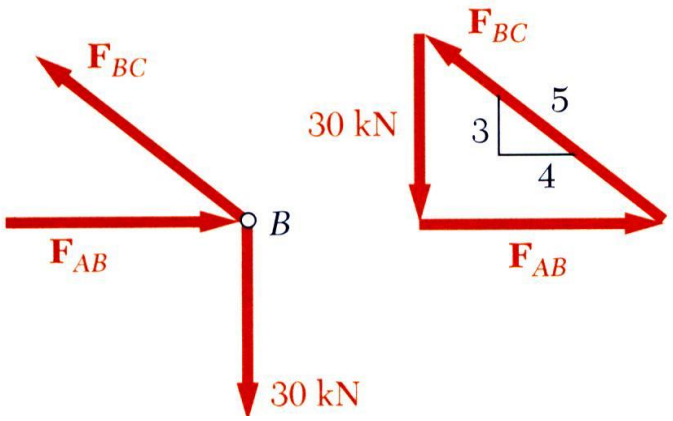


- The boom and rod are 2-force members, i.e., the members are subjected to only two forces which are applied at member ends
- For equilibrium, the forces must be parallel to an axis between the force application points, equal in magnitude, and in opposite directions
  - Joints must satisfy the conditions for static equilibrium which may be expressed in the form of a force triangle:

$$\sum \vec{F}_B = 0$$

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30\,\text{kN}}{3}$$

$$F_{AB} = 40\,\text{kN} \qquad F_{BC} = 50\,\text{kN}$$



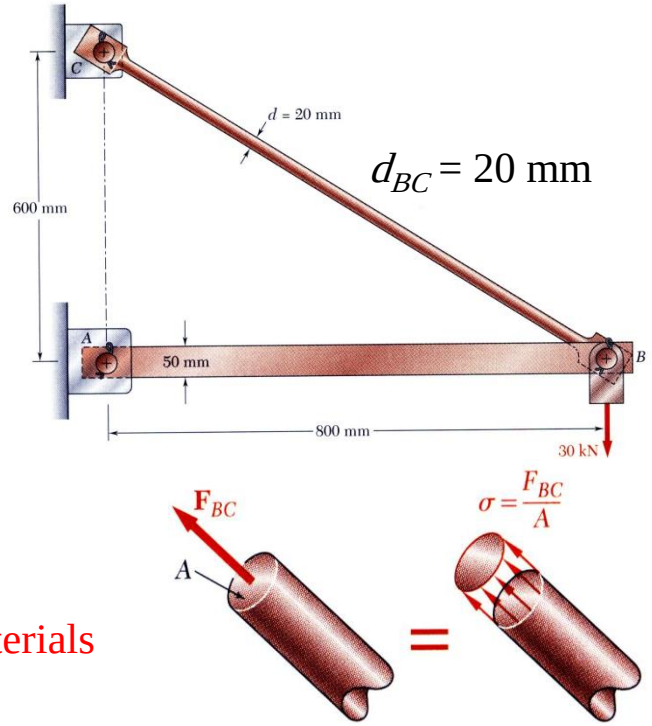
Can the structure safely support the 30 kN load?

- From a statics analysis

  F_{AB} = 40 kN (compression), F_{BC} = 50 kN (tension)

- At any section through member BC, the internal force is 50kN with a force intensity or stress of

$$\sigma_{BC} = \frac{P}{A} = \frac{50\times10^{3}\,\mathrm{N}}{\pi * 20^{2}/4\mathrm{mm}^{2}} = \frac{50000N}{314mm^{2}} = 159\ \mathrm{MPa}$$



- From the material properties for steel, the allowable stress is

$$\sigma_{\mathrm{all}} = 165\,\mathrm{MPa}$$

- **Conclusion: the strength of member *BC* is adequate**
- Design of new structures requires selection of appropriate materials and component dimensions to meet performance requirements

- **For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum (σ_{all}= 100 MPa). What is an appropriate choice for the rod diameter?**

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50\times10^{3}\,\mathrm{N}}{100\times10^{6}\,\mathrm{Pa}} = 500\times10^{-6}\,\mathrm{m}^{2}$$

$$A = \pi\frac{d^{2}}{4} \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4\left(500\times10^{-6}\,\mathrm{m}^{2}\right)}{\pi}} = 2.52\times10^{-2}\,\mathrm{m} = 25.2\,\mathrm{mm}$$

- **An aluminum rod 26 mm or more in diameter is adequate**